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# SOME PROPERTIES OF A TRIPLE SEQUENCE SPACES OF FUZZY REAL NUMBERS BY DOUBLE ORLICZ FUNCTIONS

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**Abstract**
This study uses a double Orlicz functions to provide some new triple sequence spaces. the researcher addresses to prove various algebraic and topological properties of these spaces, such as completeness, solidity, monotonicity, symmetry and convergence indepece. Moreover, This research analyze the relationships between these spaces.

**Keywords** Fuzzy numbers, Triple sequences, Orlicz function, completeness, monotone, symmetric, convergence free.

## INTRODUCTION

Zadeh established the idea of fuzzy set theory [1]. Based on this, other authors have introduced and examined sequences of fuzzy numbers, studying significant properties. In Bromwich, the work on the double sequence of real numbers was discovered [2]. In addition, a number of people, including Moricz [3], Basarir and Sonalncan [4], and others, exacted the double sequence.

$E_F^3$, denoting the family of all $\mathbb{R}$ or $\mathbb{C}$ triple sequences, is used throughout this article.

A triple sequence in [5][6] is a function from $(\mathbb{N})$to( $\mathbb{R}$ or $\mathbb{C}$ ). Using a function $\Upsilon$ where $\Upsilon = (\Upsilon_1(\mathfrak{X}).\Upsilon_2(\mathfrak{M}))$. novel outcomes of triple sequence spaces would be examined by double Orlicz function.

Assume that $(\mathfrak{X}.\mathfrak{M}) = (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\,.\ \mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$. symbolizes a triple infinite array of elements with the name $(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}).(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$. $\mathfrak{X} = (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$ be an infinite array of elements $\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}$, and $\mathfrak{M} = (\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$ be an endless array of elements $\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}$. respectively.

Prof. Wlayshaw Roman Orlicz, a Polish scholar, established the concept of the Orlicz function and gave it his name; consequently, he created the Orlicz space [7].

## 2)Definitions and preliminaries

**Definition 2.1**

Assume that the sequence $\mathfrak{X} = (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$. $\mathfrak{M} = (\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$are Cauchy, then $(\mathfrak{X}.\mathfrak{M}) = (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$ is said to be a triple Cauchy sequence if for every $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that $\bar{d}\left((\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}_{i.j.e}).(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{i.j.e})\right) < \epsilon$.for all $i \geq \mathfrak{h} \geq N$. $\mathrm{j} \geq \mathfrak{U} \geq N$. $e \geq \mathfrak{N} \geq N$ where $\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}_{i.j.e}) < \epsilon$ and $\bar{d}(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{i.j.e}) < \epsilon$ .

**Definition 2.2**

A Triple sequence space $E_F^3$ is said to be convergence free if $(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3$. whenever

$(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\ \mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3$ and $(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\ \mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) = (0.0)$ implies $(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) = (0.0)$.

**Definition 2.3**

If $(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}).(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3$ suchthat $\bar{d}\left((\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right) \leq \bar{d}\left((\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right)$ for all $\mathfrak{h}.\mathfrak{U}.\mathfrak{N} \in \mathbb{N}$. then $E_F^3$ is said to be solid.

**Definition 2.4**

A canonical pre-image of a triple sequence of $(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3$ is a triple sequence $(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3$ defined as follows:

$$(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) = \begin{cases} (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}). & \text{If } (\mathfrak{h}.\mathfrak{U}.\mathfrak{N}) \in K. \\ (\bar{0}.\bar{0}). & \text{otherwise.} \end{cases}$$

**Definition 2.5:**

If all of the canonical pre-images of a triple sequence space $E_F^3$ are included within it, then $E_F^3$ is considered monoton.

**Definition 2.6:**

For any $(\mathfrak{X}.\mathfrak{M}) = (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3$, a $E_F^3$ is considered symmetric if $G(\mathfrak{X}.\mathfrak{M}) \subseteq E_F^3$ where $G(\mathfrak{X}.\mathfrak{M})=\left\{(\mathfrak{X}_{\pi(\mathfrak{h})\pi(\mathfrak{U})\pi(\mathfrak{N})}.\mathfrak{M}_{\pi(\mathfrak{h})\pi(\mathfrak{U})\pi(\mathfrak{N})}): \pi \text{ is a permutation of } \mathbb{N}\times\ \mathbb{N}\times\mathbb{N}\right\}$.

We now define the following classes of triple sequences**:**

$(l_\infty)_F^3\Upsilon=$

$$\left\{\begin{matrix} (\mathfrak{X}_{s\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})}{\rho}\right) \vee \Upsilon_2\left(\frac{\bar{d}(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})}{\rho}\right)\right\} < \infty \\ \text{for some } \rho > 0. \end{matrix}\right\}.$$

where

$$(l_\infty)_F^3\Upsilon_1 = \left\{\begin{matrix} (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})}{\rho}\right)\right\} < \infty \\ \text{for some } \rho > 0. \end{matrix}\right\}.$$

and

$$(l_\infty)_F^3\Upsilon_2 = \left\{\begin{matrix} (\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_2\left(\frac{\bar{d}(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})}{\rho}\right)\right\} < \infty \\ \text{for some } \rho > 0. \end{matrix}\right\}.$$

$(c)_F^3\Upsilon=$

$$\left\{ \begin{array}{c} (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : lim_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} \left\{ \Upsilon_1 \left( \frac{\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\ell_1)}{\rho} \right) \vee \Upsilon_2 \left( \frac{\bar{d}(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\ell_2)}{\rho} \right) \right\} = 0 \\ \text{for some } \rho > 0. \end{array} \right\}.$$

where

$$(c)_F^3 \Upsilon_1 = \left\{ \begin{array}{c} (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : lim_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} \left\{ \Upsilon_1 \left( \frac{\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\ell_1)}{\rho} \right) \right\} = 0 \\ \text{for some } \rho > 0. \end{array} \right\}$$

and

$$(c)_F^3 \Upsilon_2 = \left\{ \begin{array}{c} (\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : lim_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} \left\{ \Upsilon_2 \left( \frac{\bar{d}(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\ell_2)}{\rho} \right) \right\} = 0 \\ \text{for some } \rho > 0. \end{array} \right\}.$$

$(c_0)_F^3 \Upsilon$=

$$\left\{ \begin{array}{c} (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : lim_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} \left\{ \Upsilon_1 \left( \frac{\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})}{\rho} \right) \vee \Upsilon_2 \left( \frac{\bar{d}(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})}{\rho} \right) \right\} = 0 \\ \text{for some } \rho > 0. \end{array} \right\}.$$

where

$$(c_0)_F^3 \Upsilon_1 \left\{ \begin{array}{c} (\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : lim_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} \left\{ \Upsilon_1 \left( \frac{\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})}{\rho} \right) \right\} = 0 \\ \text{for some } \rho > 0. \end{array} \right\}.$$

and

$$(c_0)_F^3 \Upsilon_2 \left\{ \begin{array}{c} (\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in E_F^3 : lim_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} \left\{ \Upsilon_2 \left( \frac{\bar{d}(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})}{\rho} \right) \right\} = 0 \\ \rho > 0. \end{array} \right\}.$$

Moreover, The triple sequence classes are defined by us.

$$(m)_F^3 = (c)_F^3(\Upsilon) \cap (l_\infty)_F^3(\Upsilon).$$

$$(m_0)_F^3 = (c_0)_F^3(\Upsilon) \cap (l_\infty)_F^3(\Upsilon).$$

## 3) Main Results

**Theorem 3.1**: Let $(\wp_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$ be bounded. Then the classes of triple sequences $(l_\infty)_F^3(\Upsilon . \wp)$ is complete metric spaces with respect to the distance defined by

$G((\mathfrak{X}.\mathfrak{M}).(\mathfrak{B}.\mathfrak{Q}))$=

$$inf\left\{\rho^{\frac{\wp_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}}{\mathcal{J}}} > 0: sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\left(\Upsilon_1\left(\frac{\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})}{\rho}\right)\right) \vee \left(\Upsilon_2\left(\frac{\bar{d}(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})}{\rho}\right)\right)\right\} \le 1\right\}$$

$$\mathcal{J} = \max(1.2^{T-1})$$

where

$$G(\mathfrak{X}.\mathfrak{M}) = inf\left\{\rho^{\frac{\wp_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}}{\mathcal{J}}} > 0: sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left(\Upsilon_1\left(\frac{\bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})}{\rho}\right)\right) \le 1\right\}$$

$$G(\mathfrak{B}.\mathfrak{Q}) = inf\left\{\rho^{\frac{\wp_{\mathfrak{s}.\mathfrak{r}.\mathcal{v}}}{\mathcal{J}}} > 0: sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left(\Upsilon_2\left(\frac{\bar{d}(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})}{\rho}\right)\right) \le 1\right\}$$

**Proof :** Let us consider the case $(l_\infty)_F^3(\Upsilon.\wp)$ and the other cases can be established next similar techniques..

Let $(\mathfrak{X}^i).(\mathfrak{M}^i)$ be any Cauchy sequences in $(l_\infty)_F^3(\Upsilon_1.\wp)$. $(l_\infty)_F^3(\Upsilon_2.\wp)$ respectively, hence $(\mathfrak{X}^i.\mathfrak{M}^i) = (\mathfrak{X}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$ be a triple Cauchy sequence

Let $\epsilon > 0$, $\mathfrak{X}_0.r > 0$ be fixed. Then ∀ there exists a positive integer N such that $G_{\Upsilon_1}(\mathfrak{X}^i.\mathfrak{X}^j) < \frac{\epsilon}{r\mathfrak{X}_0}$ and $G_{\Upsilon_2}(\mathfrak{M}^i.\mathfrak{M}^j) < \frac{\epsilon}{r\mathfrak{X}_0}$. for $i.j \ge N$. and consequently,

$G_\Upsilon\left((\mathfrak{X}^i.\mathfrak{X}^j).(\mathfrak{M}^i.\mathfrak{M}^j)\right) = \left(G_{\Upsilon_1}(\mathfrak{X}^i.\mathfrak{X}^j).G_{\Upsilon_2}(\mathfrak{M}^i.\mathfrak{M}^j)\right) < \frac{\epsilon}{r\mathfrak{X}_0}$,

for all $i.j \ge N$.

By definition of $G$. we obtain

$$inf\left\{\rho > 0: sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}\left(\mathfrak{X}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{\rho}\right) \vee \Upsilon_2\left(\frac{\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{\rho}\right)\right\} \le 1\right\}.$$

Thus,

$$sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}\left(\mathfrak{X}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{\rho}\right) \vee \Upsilon_2\left(\frac{\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{\rho}\right)\right\} \le 1.$$

for all $i.j \ge N$.

$$\Longrightarrow sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}\left(\mathfrak{X}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{G_{\Upsilon_1}\left(\mathfrak{X}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}\right) \vee \Upsilon_2\left(\frac{\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{G_{\Upsilon_2}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}\right)\right\} \le 1.$$

for each $i.j \ge N$.

Since $\wp_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}$bounded it follows that

$$\left\{\Upsilon_1\left(\frac{\bar{d}\left(\mathfrak{X}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{G_{\Upsilon_1}\left(\mathfrak{X}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}\right) \vee \Upsilon_2\left(\frac{\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{G_{\Upsilon_2}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}\right)\right\} \le 1.$$

for each $\mathfrak{s}.\mathfrak{r}.\mathcal{v} \ge 1$ and for all $i.j \ge N$.

Hence one can find $r > 0$ with $\Upsilon_1\left(\frac{r\mathfrak{X}_0}{2}\right) \ge 1$ and $\Upsilon_2\left(\frac{r\mathfrak{X}_0}{2}\right) \ge 1$, such that

$$\Upsilon_1\left(\frac{\bar{d}\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{G_{\Upsilon_1}\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}\right)\leq \Upsilon_1\left(\frac{r\mathfrak{X}_0}{2}\right) \text{and } \Upsilon_2\left(\frac{\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{G_{\Upsilon_2}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}\right)\leq \Upsilon_2\left(\frac{r\mathfrak{X}_0}{2}\right)$$

Hence, $\Upsilon\left(\frac{r\mathfrak{x}_0}{2}.\frac{r\mathfrak{x}_0}{2}\right) = \left(\Upsilon_1\left(\frac{r\mathfrak{x}_0}{2}\right).\Upsilon_2\left(\frac{r\mathfrak{x}_0}{2}\right)\right) \geq (1.1)$ therefore,

$$\left\{\Upsilon_1\left(\frac{\bar{d}\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{G_{\Upsilon_1}\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}\right).\Upsilon_2\left(\frac{\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{G_{\Upsilon_2}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}\right)\right\}\leq\left(\Upsilon_1\left(\frac{r\mathfrak{X}_0}{2}\right).\Upsilon_2\left(\frac{r\mathfrak{X}_0}{2}\right)\right).$$

This implies that.

$\bar{d}\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)\leq\frac{r\mathfrak{x}_0}{2}\cdot G_{\Upsilon_1}\left((\mathfrak{x}^i.\mathfrak{x}^j)\right)$ for all $i.\jmath \geq n_0$ .

$$\bar{d}\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)\leq\frac{r\mathfrak{x}_0}{2}\cdot\frac{\epsilon}{r\mathfrak{x}_0}=\frac{\epsilon}{2}\qquad i.\jmath\geq n_0.$$

$$\Rightarrow \bar{d}\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)\leq\frac{\epsilon}{2}\qquad i.\jmath\geq n_0.$$

and

$$\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)\leq\frac{r\mathfrak{x}_0}{2}\cdot G_{\Upsilon_2}\left((\mathfrak{M}^i.\mathfrak{M}^j)\right)\ i.\jmath\geq n_0.$$

$$\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)\leq\frac{r\mathfrak{x}_0}{2}\cdot\frac{\epsilon}{r\mathfrak{x}_0}=\frac{\epsilon}{2}\qquad i.\jmath\geq n_0$$

$$\Rightarrow \bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)\leq\frac{\epsilon}{2}\qquad i.\jmath\geq n_0. \text{ then}$$

$$\bar{d}\left(\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right).(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})\right)\leq\frac{r\mathfrak{x}_0}{2}.\frac{\epsilon}{r\mathfrak{x}_0}=\frac{\epsilon}{2}\ i.\jmath\geq n_0.$$

Hence $(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$ is a triple Cauchy sequence in $R^3$.

Thus,

For each $(0<\epsilon<1)$,there exists a positive integer $N$ such that $\bar{d}\left((\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X}).(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M})\right)<\epsilon$ for all $i.j\geq N$. where $\bar{d}(\mathfrak{x}^i.\mathfrak{X})<\epsilon$ and $\bar{d}\ (\mathfrak{M}^i.\mathfrak{M})<\epsilon$ for all $i.j\geq N$.

Taking $\jmath\to\infty$ and fixing $i$. so by using the continuity of $\Upsilon=(\Upsilon_1.\Upsilon_2)$ we get $sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}\left(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\lim_{j\to\infty}\mathfrak{x}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{\rho}\right)\vee\Upsilon_2\left(\frac{\bar{d}\left(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\lim_{j\to\infty}\mathfrak{M}^j_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)}{\rho}\right)\right\}\leq 1$

Thus,

$$sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X})}{\rho}\right)\vee\Upsilon_2\left(\frac{\bar{d}(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M})}{\rho}\right)\right\}\leq 1.$$

On taking the infimum of such $\rho$' s, we get,

$$inf\left\{\rho>0: sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}(\mathfrak{x}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{X})}{\rho}\right)\vee\Upsilon_2\left(\frac{\bar{d}(\mathfrak{M}^i_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M})}{\rho}\right)\right\}\leq 1\right\}\leq\epsilon$$

for all $i\geq N$ and $j\to\infty$.

Since $(\mathfrak{x}^i.\mathfrak{M}^i)\in(l_\infty)^3_F(\Upsilon.\wp)$ and $\Upsilon$ is continuous, it follows that $(\mathfrak{X}.\mathfrak{M})\in(l_\infty)^3_F(\Upsilon.\wp)$ .

**Proposition 3.1:** The class of triple sequences $(l_\infty)^3_F(\Upsilon)$ is symmetric but the class of triple sequences $(c)^3_F(\Upsilon).(c_0)_F(\Upsilon)$, are not symmetric.

**Proof:**

Noticeably the class of triple sequence $(l_\infty)_F^3(\Upsilon)$ is symmetric. However, other the class of triple sequences, could be indicated by the example below.

**Example 3.1 :** Let's say the class of triple sequences $(c)_F^3\,(\Upsilon)$ . Consider $\Upsilon\,(\mathfrak{X}.\mathfrak{M}) = (\mathfrak{X}.\mathfrak{M})$ and suppose the triple sequence $\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)$ be defined by

$\mathfrak{p}$

where

$$(\mathfrak{X}_{1\mathfrak{U}})(\mathfrak{p}) = \begin{cases} (\mathfrak{p}+1). & \text{for} -1 \le \mathfrak{p} \le 0; \\ (-\mathfrak{p}+1). & \text{for} \quad 0 \le \mathfrak{p} \le 1; \\ 0. & \text{otherwise,} \end{cases}$$

and

$$(\mathfrak{M}_{1\mathfrak{U}})(\mathfrak{p}) = \begin{cases} (\mathfrak{p}+1). & \text{for} -1 \le \mathfrak{p} \le 0; \\ (-\mathfrak{p}+1). & \text{for} \quad 0 \le \mathfrak{p} \le 1; \\ 0. & \text{otherwise.} \end{cases}$$

For $\mathfrak{h} > 1$, we have

$$\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p}) = \begin{cases} (\mathfrak{p}+2.\mathfrak{p}+2) & \text{for} -2 \le \mathfrak{p} \le -1; \\ (-\mathfrak{p}.-\mathfrak{p}) & \text{for} \ -1 \le \mathfrak{p} \le 0; \\ (0.0) & \text{otherwise.} \end{cases}$$

where

$$\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p}) = \begin{cases} \mathfrak{p}+2. & \text{for} -2 \le \mathfrak{p} \le -1; \\ -\mathfrak{p}. & \text{for} \ -1 \le \mathfrak{p} \le 0; \\ 0. & \text{otherwise.} \end{cases}$$

and

$$\left(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p}) = \begin{cases} \mathfrak{p}+2. & \text{for} -2 \le \mathfrak{p} \le -1; \\ -\mathfrak{p}. & \text{for} \ -1 \le \mathfrak{p} \le 0; \\ 0. & \text{otherwise.} \end{cases}$$

Let $\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)$ ,$\left(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)$ be a rearrangement of $\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)$ , $\left(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)$ respectively which is defined by

$$\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{h}}\right)(\mathfrak{p}) = \begin{cases} \mathfrak{p}+1. & \text{for} -1 \le \mathfrak{p} \le 0; \\ -\mathfrak{p}+1. & \text{for } 0 \le \mathfrak{p} \le 1; \\ 0. & \text{otherwise.} \end{cases}$$

and

$$\left(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{h}}\right)(\mathfrak{p}) = \begin{cases} \mathfrak{p}+1. & \text{for} -1 \le \mathfrak{p} \le 0\ ; \\ -\mathfrak{p}+1. & \text{for } 0 \le \mathfrak{p} \le 1; \\ 0. & \text{otherwise.} \end{cases}$$

Therefore, $\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{h}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{h}}\right)$ can be defined by

$$\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{h}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{h}}\right)(\mathfrak{p}) = \begin{cases} (\mathfrak{p}+1.\mathfrak{p}+1). & \text{for} -1 \le \mathfrak{p} \le 0; \\ (-\mathfrak{p}+1.-\mathfrak{p}+1). & \text{for } 0 \le \mathfrak{p} \le 1; \\ (0.0). & \text{otherwise.} \end{cases}$$

and for $\mathfrak{h} \neq \mathfrak{U}$. we have

$(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})(\mathfrak{p})=$

$$\begin{Bmatrix} (\mathfrak{p}+2\,.\,\mathfrak{p}+2). & \text{for } -2 \le \mathfrak{p} \le -1. \\ (-\mathfrak{p}\,.\,-\mathfrak{p}). & \text{for } -1 \le \mathfrak{p} \le -1. \\ (0\ .\ 0). & \text{otherwise.} \end{Bmatrix}$$

where

$$(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})(\mathfrak{p}) = \begin{cases} \mathfrak{p}+2. & \text{for } -2 \le \mathfrak{p} \le -1; \\ -\mathfrak{p}. & \text{for } -1 \le \mathfrak{p} \le 0; \\ 0. & \text{otherwise.} \end{cases}$$

and

$$(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})(\mathfrak{p}) = \begin{cases} \mathfrak{p}+2. & \text{for } -2 \le \mathfrak{p} \le -1; \\ -\mathfrak{p}. & \text{for } -1 \le \mathfrak{p} \le 0; \\ 0. & \text{otherwise.} \end{cases}$$

Thus,
$(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in (c)_F^3(\Upsilon)$ but $(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \notin (c)_F^3(\Upsilon)$.Hence $(c)_F^3(\Upsilon)$ is not symmetric. In same sense, it could be indicated that other spaces of triple sequences are not symmetric too.

**Proposition 3.2:** The classes of triple sequences $(l_\infty)_F^3(\Upsilon).\,(c_0)_F^3(\Upsilon)$ and $(m_0)_F^3(\Upsilon)$ are solid**.**

**Proof :** Consider $(l_\infty)_F^3\,(\Upsilon)$ the class of triple sequences.

So $(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}) \in (l_\infty)_F^3\,(\Upsilon)$ and $(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}})$ be such that.

$$\bar{d}(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}) \le \bar{d}(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})$$

and

$$\bar{d}(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}) \le \bar{d}(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})$$

and consequently

$$\bar{d}\left((\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right) \le \bar{d}\left((\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right)$$

as $\Upsilon = (\Upsilon_1.\Upsilon_2)$ is increasing , we have

$$sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_1\left(\frac{\bar{d}\left((\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right)}{\rho}\right)\right\} \le sup_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\left\{\Upsilon_2\left(\frac{\bar{d}\left((\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right)}{\rho}\right)\right\}$$

Hence, the classes of triple sequences $(l_\infty)_F^3(\Upsilon)$ is solid. In same way, we could recognize other spaces are solid too by following same sense .■

**Proposition 3.3:** The classes of triple sequences$(c)_F^3(\Upsilon).\,(m)_F^3(\Upsilon)$ are not monotone and hence not solid**.**

**Corollary 3.1** $Z(\Upsilon_1) \cap Z(\Upsilon_2) \subseteq Z(\Upsilon_1 + \Upsilon_2)$. for $Z = (l_\infty)_F^3(\Upsilon).\,(c)_F^3(\Upsilon)$

**Corollary 3.2:** Let $\Upsilon$ and $\Upsilon_1$be two Orlicz function then $Z(\Upsilon_1) \subseteq Z(\Upsilon \circ \Upsilon_1)$. for $Z =$

$(l_\infty)_F^3 . (c)_F^3 . (c_0)_F^3 . (m)_F^3$. and $(m_0)_F^3$.

**Proof :** The result will be proven for the case $Z = (c_0)_F^3$. The other cases can be prove by using the same technique. Take $\epsilon > 0$. there exists $n > 0$. such that $\epsilon = \Upsilon(n)$. Let $\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} . \mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) \in Z(\Upsilon_1)$. then, there exist $k_0 . l_0 \in \mathbb{N}$. such that

$$\Upsilon_1\left[\frac{\bar{d}\left((\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right)}{\rho}\right] < n. \text{ for some } \rho > 0.$$

$$\text{Let } \left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) = \Upsilon_1\left[\frac{\bar{d}\left((\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right)}{\rho}\right]. \text{ for some } \rho > 0.$$

Since $\Upsilon$ is continuous and non-decreasing, we get

$$\Upsilon\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) = \Upsilon\left[\Upsilon_1\left[\frac{\bar{d}\left((\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0}).(\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\bar{0})\right)}{\rho}\right]\right] < \Upsilon(n) = \epsilon.$$

for some $\rho > 0$.

Which implies that, $\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} . \mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) \in Z(\Upsilon \circ \Upsilon_1)$. ■

**Proposition 3.4** The class of triple sequences $(l_\infty)_F^3(\Upsilon).(c)_F^3(\Upsilon).(c_0)_F^3(\Upsilon).(m)_F^3(\Upsilon)$ are not convergent free.

**Proof:** The following Example will lead to such result.

**Example 3.2:**

Consider the classes of triple sequences $(c)_F^3(\Upsilon)$. Suppose $\Upsilon(\mathfrak{X}.\mathfrak{M}) = (\mathfrak{X}.\mathfrak{M})$ and consider the triple sequence $\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} . \mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)$ defined by $(\mathfrak{X}_{1\mathfrak{U}} . \mathfrak{M}_{1\mathfrak{U}}) = (\bar{0}.\bar{0})$ and for other values,

$$\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}} . \mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p}) = \begin{cases} (1.1). & \text{for } 0 \le \mathfrak{p} \le 1; \\ (-\mathfrak{h}\mathfrak{p}(\mathfrak{h}+1)^{-1} + (2\mathfrak{h}+1)(\mathfrak{h}+1)^{-1}.-\mathfrak{h}\mathfrak{p} + (2\mathfrak{h}+1)(\mathfrak{h}+1)^{-1}). & \text{for } 1 \le \mathfrak{p} \le 2+\mathfrak{h}^{-1}; \\ (0.0). & \text{otherwise.} \end{cases}$$

where

$$\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p}) = \begin{cases} 1. & \text{for } 0 \le \mathfrak{p} \le 1; \\ -\mathfrak{h}\mathfrak{p}(\mathfrak{h}+1)^{-1} + (2\mathfrak{h}+1)(\mathfrak{h}+1)^{-1}. & \text{for} 1 \le \mathfrak{p} \le 2+\mathfrak{h}^{-1}. \\ 0. & \text{otherwise.} \end{cases}$$

and

$$\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p}) = \begin{cases} 1. & \text{for } 0 \le \mathfrak{p} \le 1; \\ -\mathfrak{h}\mathfrak{p}(\mathfrak{h}+1)^{-1} + (2\mathfrak{h}+1)(\mathfrak{h}+1)^{-1}. & \text{for} 1 \le \mathfrak{p} \le 2+\mathfrak{h}^{-1}. \\ 0. & \text{otherwise.} \end{cases}$$

Let the triple sequence $\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)$. be defined by $(\mathfrak{B}_{1\mathfrak{U}.}.\mathfrak{Q}_{1\mathfrak{U}}) = (\bar{0}.\bar{0})$. and for other values, $\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)$ can be defined as

$\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p})$=

$$\begin{cases} (1.1). & \text{for } 0 \le \mathfrak{p} \le 1; \\ \left((\mathfrak{h}-\mathfrak{p})(\mathfrak{h}-1)^{-1}.(\mathfrak{h}-\mathfrak{p})(\mathfrak{h}-\mathfrak{p})^{-1}\right). & \text{for } 1 \le \mathfrak{p} \le \mathfrak{h} \\ (0.0). & \text{otherwise.} \end{cases}$$

where

$$\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p}) = \begin{cases} 1. & \text{for } 0 \le \mathfrak{p} \le 1; \\ (\mathfrak{h}-\mathfrak{p})(\mathfrak{h}-\mathfrak{p})^{-1}. & \text{for } 1 \le \mathfrak{p} \le \mathfrak{h}; \\ 0. & \text{otherwise.} \end{cases}$$

and

$$\left(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right)(\mathfrak{p}) = \begin{cases} 1. & \text{for } 0 \le \mathfrak{p} \le 1; \\ (\mathfrak{h}-\mathfrak{p})(\mathfrak{h}-\mathfrak{p})^{-1}. & \text{for } 1 \le \mathfrak{p} \le \mathfrak{h}; \\ 0. & \text{otherwise.} \end{cases}$$

Then $\left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) \in (c)_F^3(\Upsilon)$. and $(\mathfrak{M}_{s.t.a}) \in (c)_F^3(\Upsilon)$.

$\Longrightarrow \left(\mathfrak{X}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{M}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) \in (c)_F^3(\Upsilon)$ but $\left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) \notin (c)_F^3(\Upsilon)$. and $\left(\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) \notin (c)_F^3(\Upsilon)$.

$\Longrightarrow \left(\mathfrak{B}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}.\mathfrak{Q}_{\mathfrak{h}.\mathfrak{U}.\mathfrak{N}}\right) \notin (c)_F^3(\Upsilon)$.

Therefore, $(c)_F^3(\Upsilon)$ isn't convergent free. Likewise, the other spaces are also not convergent free.

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