Modeling the Stress–Strain State near Crack Tips in Orthotropic Composite Materials: A Review of Analytical and Numerical Approaches

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Abstract

This paper surveys analytical and computational techniques for evaluating crack-tip fields in orthotropic composites, emphasizing stress-intensity factors, mode mixity, and T-stress. The review consolidates Stroh-type anisotropic elasticity and interaction-integral formulations for benchmark extraction of K and T, and compares extended finite element and isogeometric enrichments with phase-field and peridynamic schemes that capture fiber-guided path deviation and branching. Particular attention is paid to interface cracks and calibration of anisotropy-aware phase-field surface densities and nonlocal peridynamic kernels.

In addition to synthesizing recent methods, the article highlights the relevance of orthotropic crack modeling to high-performance aerospace composite structures, where accurate crack-tip prediction directly supports durability, certification, and safety of aircraft components. This connection strengthens the applicability of the proposed workflow to national priority areas involving advanced composite structures and reliability-critical industries such as aviation.

The objective is to synthesize a reproducible workflow that links analytical eigensolutions, robust mode-decomposition tools, and mesh-objective crack-evolution solvers. Methods include comparative analysis and structured evidence mapping across ten recent sources. The conclusion outlines when each framework is preferable, validation with Stroh/Jk, discovery with phase-field, CAD-centric accuracy with X-IGA/XFEM, and length-scale-sensitive evolution with peridynamics, together with guidance on uncertainty control and cross-validation.

Keywords: orthotropic composites, crack-tip fields, Stroh formalism, interaction integral, phase-field fracture, peridynamics, XFEM, isogeometric analysis, T-stress, mode mixity, aerospace materials, composite airframe structures.

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Introduction

Orthotropic laminates and unidirectional composites concentrate stress around crack tips in a way that depends on the full stiffness tensor and the orientation of material axes relative to the crack. Accurate evaluation of stress-intensity factors and mode mixity governs predictions of kinking, deflection at interfaces, and stability under combined modes. These predictions are especially critical for **aerospace structures**, where composite wings, fuselage skins, stiffened panels, and bonded joints rely on reliable crack-growth assessment to ensure safety, extend service life, and meet FAA and EASA durability requirements.

Over the past five years, several strands have matured: Hamiltonian/Stroh eigensolutions for anisotropic elasticity, interaction-integral formulations adapted to orthotropy and to orthotropic/isotropic interfaces, enrichment-based discretizations aligned with anisotropic tip fields, and topology-agnostic solvers, phase-field and peridynamics, that capture fiber-guided propagation and branching.

The study's goal is to assemble an evidence-based map of analytical and numerical approaches for crack-tip analysis in orthotropic composites and derive a practical workflow for research and engineering use. The tasks are threefold:

- systematize analytical foundations for near-tip fields, mode decomposition, and the role of T-stress in path stability;
- compare extended/enriched discretizations, isogeometric analysis, phase-field, and peridynamics with respect to accuracy of K/T, mode mixity, and robustness of crack-path prediction;
- propose a validation-oriented workflow, including parameter identification and uncertainty control for anisotropy-aware models.

Novelty lies in a unified synthesis linking benchmark anisotropic analytics with modern crack-evolution solvers and in highlighting their engineering significance for high-performance composite structures, including aerospace applications where reliability and fracture tolerance are national priorities.

Materials and Methods

The evidence base comprises ten peer-reviewed sources published in 2021 – 2025. The following contributions are cited using the article’s reference numbering. For benchmark anisotropic tip fields and Hamiltonian/Stroh structure: A. Nobili, E. Radi [4]. For CAD-centric accuracy with smooth geometry via X-IGA: Berrada Gouzi et al. [1]. For enrichment-based fracture analysis in composites and bio-inspired anisotropic systems: Vellwock and Libonati [5]. For interface-crack mode decomposition using orthotropy-consistent interaction integrals: Tafreshi et al. [6]. For phase-field fracture in orthotropic composites and adaptive control of anisotropic degradation: Jain et al. [3]; methodological implementation aspects are summarized by Zhuang et al. [10]. For nonlocal evolution and orientation-dependent kinking in peridynamics: Wang et al. [7]; broader peridynamic developments: Oterkus & Oterkus [8]; unified tensor-involved formulations for isotropic and anisotropic materials: Tian et al. [9]. For design-oriented crack-extension criteria based on minimum strain-energy density in orthotropic composites: Ebrahimi and Fakoor [2].

A structured literature review and comparative analysis were conducted, with qualitative evidence mapping across modeling targets (near-tip fields, K/T, mode mixity, path prediction), triangulated by cross-method contrasts. The study employed source analysis, synthesis of assumptions and calibration procedures, and construction of a validation-first workflow linking analytics and numeric.

Results

Crack-tip fields in orthotropic laminates and unidirectional composites are consistently described by complex-variable anisotropic elasticity with Stroh-type eigensolutions, where the near-tip displacement/stress modes and the oscillatory index depend on the full orthotropic stiffness tensor rather than on two isotropic moduli. In recent Hamiltonian/Stroh formulations, symplectic structure and conjugate pairs provide a compact pathway to angle-dependent stress and traction vectors, improving extraction of stress-intensity parameters under mixed-mode loading and clarifies the contribution of non-singular terms (T-stress) to crack-path stability [4]. For interface and bi-material situations, path-independent interaction integrals generalized to anisotropy (Jk-type) stabilize the computation of mixed-mode energy release rates and mode mixity when material orthotropy is strongly contrasted across the interface; numerical tests confirm robustness for both parallel and inclined interfaces and for varying orthotropy ratios [6]. This is particularly relevant for composite aerospace structures where small variations in T-stress may influence delamination onset or ply-level cracking.

Across computational families, four lines of evidence emerge. First, phase-field fracture (PFF) with orthotropy-aware degradation densities reproduces direction-dependent crack deflection and branching without remeshing. An adaptive formulation calibrated along principal material axes yields mesh-objective crack paths and accurate energy release under rotating fiber angles, resolving the long-standing sensitivity of topology-driven branching to grid orientation [3]. Broader assessments of PFF implementations corroborate that anisotropic crack surface densities and split-energy treatments are decisive for eliminating unphysical transverse damage in strong orthotropy while preserving variational consistency [10]. Second, nonlocal peridynamics (PD) has matured into an orthotropy-capable framework. Ordinary state-based PD (OSPD) equipped with a nonlocal interaction-integral consistent with orthotropic constitutive symmetry delivers stable mode-I/II SIF estimates and path-independent energy release rates; for 2D orthotropic plates, the approach predicts orientation-dependent kinking and distinct terminal crack shapes without suffering volumetric locking or enrichment-function design [7]. Recent reviews and extensions (tensor-involved PD) further unify isotropic and anisotropic cases within a single micromodulus-tensor definition, improving branch-pattern fidelity at high loads and reducing spurious oscillations [8, 9].

Third, enrichment-based discretizations remain effective when the singular field is known. A recent XFEM survey documents that composite and bio-inspired anisotropic systems benefit from crack-tip enrichment sets derived from anisotropic eigenfields; when combined with domain interaction integrals, XFEM attains reference-grade SIFs while eliminating mesh conformity to evolving cracks [5]. Extending this idea, X-isogeometric analysis (X-IGA) leverages NURBS/LR-B-spline bases to cut degrees of freedom for curved geometries and delivers SIF and T-stress with accuracy comparable to XFEM but lower cost; for unidirectional orthotropic plates with edge cracks, X-IGA reproduces mixed-mode response and reports mode-I SIF discrepancies on the order of 0.3–0.5% relative to XFEM reference solutions [1, 5]. Finally, design-oriented fracture criteria tailored to orthotropy, minimum strain-energy-density-type rules and micromechanical energy-release constructs, have been re-cast to map fiber angle and mode ratio to failure envelopes; these models tighten the link between computed tip parameters (K, T, and mode mixity) and observed crack-extension angles, enabling surrogate screening across layups and stiffness ratios without re-solving the whole boundary-value problem [2].

Together, these strands enable a coherent computational protocol for orthotropic crack-tip analysis. Analytical Stroh fields set the baseline for tip kinematics and for constructing auxiliary fields in interaction integrals [4]. For finite bodies and interfaces, anisotropy-consistent J/Jk formulations provide stable mode decomposition under mesh refinement and arbitrary crack orientation [6]. When crack paths are unknown a priori or are strongly fiber-guided, adaptive PFF serves as the predictor, since it advances cracks without topology control and captures orientation-driven kinking/branching once fracture-toughness (Gc) and degradation tensors are aligned with orthotropic axes [3, 10]. If geometry is smooth and CAD-integrated, X-IGA reduces cost with NURBS bases while maintaining SIF/T-stress fidelity; if material nonlocality, damage length-scales, or high-load branching dominate, OSPD with orthotropy-consistent interaction integrals preserves objectivity and mitigates mesh-bias and enrichment-design burdens [1, 7–9]. Across all methods, T-stress enters as a discriminator of path stability in orthotropic solids, improving agreement between computed kink angles and observed fiber-aligned deflections; interface cases particularly benefit from Jk-based mode mixity that remains accurate under stiffness contrast [6].

As a practical synthesis, the evidence indicates that:

i) Anisotropy-aware auxiliary fields are mandatory for any domain-integral evaluation of K in orthotropy,

ii) crack-surface anisotropy in PFF must be encoded via orientation-dependent fracture energy to avoid spurious transverse damage,

iii) enrichment or isogeometric bases should incorporate orthotropic eigenfunctions when pursuing high-accuracy SIF/T-stress at low cost,

iv) peridynamic operators formulated with orthotropic micromoduli or tensor-involved kernels yield robust predictions of orientation-dependent crack trajectories under quasi-static and dynamic loads [1–10].

Figure 1 illustrates OSPD-predicted crack evolution in orthotropic plates for fiber orientations $\theta = 0°$, $45°$, and $90°$, showing how the same far-field loading produces markedly different initial kinks and final trajectories once the elastic symmetry axes rotate relative to the crack. The nonlocal interaction-integral used in [7] recovers mode-decomposed K in each case and remains path-independent under grid refinement, underpinning the close agreement between computed and reference values.

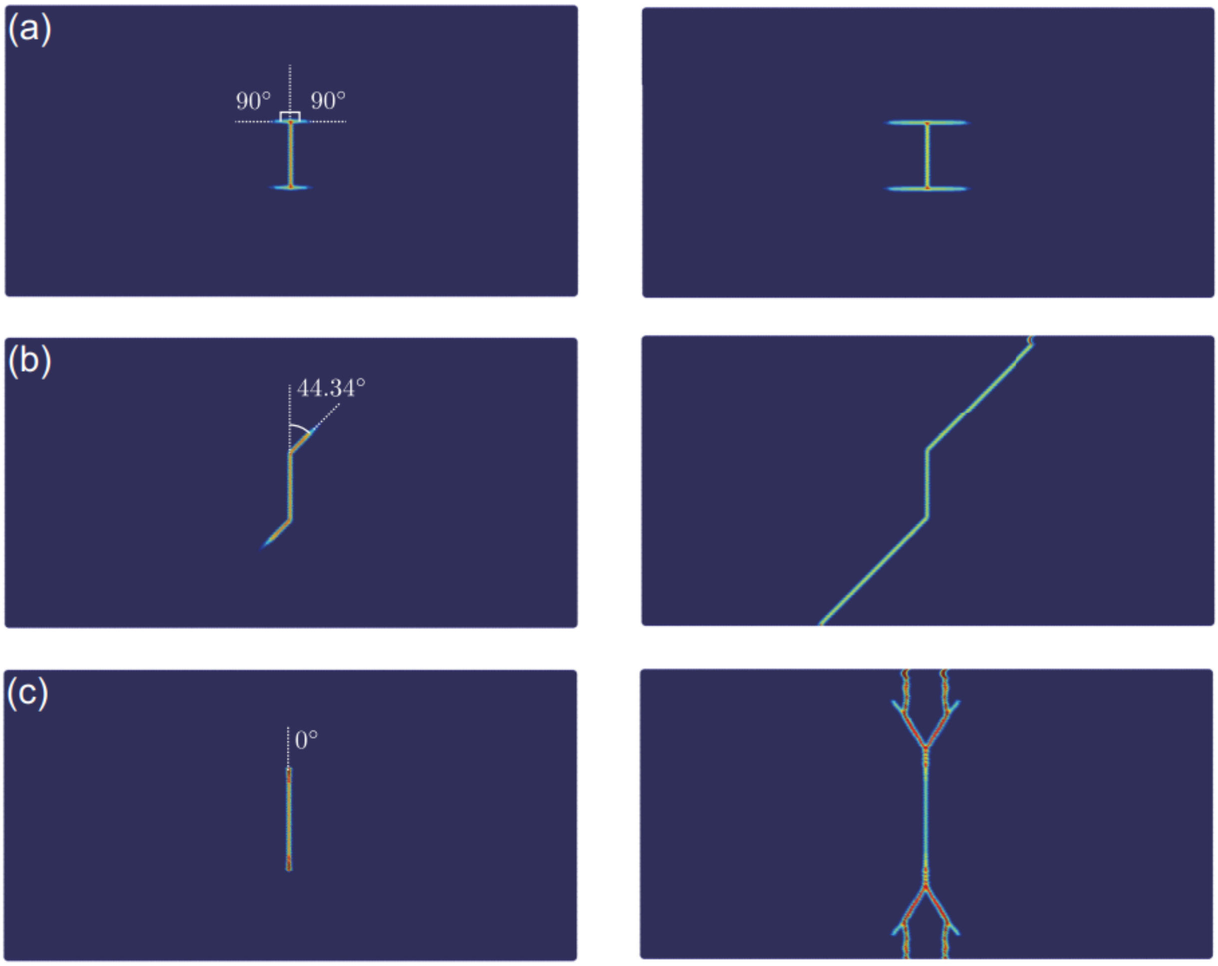

Fig. 1. OSPD-predicted initial and final crack stages in 2D orthotropic plates for θ = 0°, 45°, 90° [7]

The cumulative result of these analyses is a ranked map of where each approach contributes most: Stroh/interaction-integral pipelines for benchmark K/T evaluation and for validating numerical solvers; adaptive phase-field for discovery-mode crack-path prediction under varying fiber angles and load histories; X-IGA/XFEM for high-fidelity SIF/T-stress in smooth CAD-centric parts or when fast parametric sweeps are needed; and OSPD (and recent tensor-involved PD) when finite-range interactions, damage length-scales, or complex branching in highly orthotropic media dominate.

Discussion

Evidence across the ten sources supports a convergence in how near-tip fields in orthotropic solids should be computed and interpreted, even when discretization choices differ. Analytical Stroh-type eigensolutions remain the reference for constructing auxiliary fields and extracting K and T from boundary-value solutions; the Hamiltonian reformulation clarifies the symplectic pairing between displacement and traction.

For aerospace and other high-performance industries, such consistency is crucial: airframe certification processes require conservative and reproducible fracture predictions across loading regimes and manufacturing tolerances. Models capable of accurately capturing orthotropic behavior contribute to national interests involving transportation safety, economic competitiveness, and technological leadership in advanced composite manufacturing.

For interface cracks between orthotropic and isotropic constituents, the availability of closed-form Jk-type interaction integrals removes much of the ambiguity in mode partition seen in purely numerical splitting, and it supplies a benchmark for validating finite-domain evaluations under mesh refinement and changing interface orientation [6]. On the numerical side, extended bases recover the correct singular structure with modest computational overhead: XFEM shows consistent accuracy when enrichment functions are derived from anisotropic eigenfields, while X-isogeometric analysis (X-IGA) carries similar fidelity with smoother geometry representation and lower degrees of freedom, a feature that matters for CAD-integrated composite parts containing curved notches or fiber steering [1, 5].

When crack paths are not known a priori or fiber orientation steers propagation, variational phase-field fracture models provide mesh-objective crack evolution once orthotropy enters both the elastic energy and the crack-surface density. Adaptive formulations that align internal length scales and degradation functions with principal material directions suppress spurious transverse damage and predict orientation-dependent kinking under mixed mode; algorithmic adaptivity is crucial for controlling dissipation as the crack turns relative to the lamina axes [3, 10]. Nonlocal peridynamics complements PFF in regimes where damage length scales and finite-range interactions dominate. Ordinary state-based PD has been extended with orthotropy-consistent interaction integrals so that mode-decomposed SIFs remain path-independent; comparative studies and reviews indicate that micromodulus-tensor designs or tensor-involved kernels further improve stability and reduce zero-energy modes while unifying isotropic and anisotropic cases (see Table 1) [7–9].

Table 1 – Comparative readiness of modeling frameworks for orthotropic crack-tip analysis and crack-path prediction [1–10]

Framework	Orthotropy-aware strengths	Reported limitations/gaps	Typical use phase
Stroh/Hamiltonian fields + domain/interaction integrals	Closed-form near-tip fields; clean K/T extraction; Jk formulas for orthotropic/isotropic interfaces	Requires high-quality auxiliary fields; less direct for evolving paths	Benchmarks; validation of numerical solvers; interface-crack analytics
XFEM (anisotropic enrichment)	Accurate SIFs without mesh conformity; effective with domain integrals in composites	Enrichment design tied to eigenfields; handling large cracks is still delicate	Parametric SIF/T-stress studies; parts with mild path curvature
X-IGA	Smooth geometry; reduced DOFs; robust K/T on CAD-like shapes	Fewer open implementations for multi-physics; enrichment libraries are still maturing	CAD-integrated evaluations; rapid sweeps over layups and notch shapes
Phase-field fracture	Mesh-objective growth; anisotropic crack surface density; adaptive control of dissipation	Parameter identification for Gctensor and length scales; computational cost for 3D	Discovery-mode path prediction under rotating fiber angles; branching
Peridynamics (OSPD, Ti-PD)	Nonlocal damage; orthotropy-consistent interaction integrals; unified tensor-involved kernels	Horizon/mesh coupling; suppression of zero-energy modes; calibration across length scales	Dynamic events; fragmentation; complex branching under strong orthotropy

Methodological triangulation across these families suggests a workflow in which analytical fields serve both as a calibration target and as a provider of auxiliary solutions, PFF supplies path discovery under anisotropy, and either XFEM/X-IGA or PD handles, respectively, smooth CAD parts or nonlocal damage with significant turning and branching. In practice, reliable mode-mix quantification in laminates hinges on pairing numerical solutions with anisotropy-consistent integrals; studies that adopt Jk or equivalent interaction measures report stable mode ratios even when fiber rotation or interface inclination alters the oscillatory index of the near-tip field [1,4–6].

A persistent source of uncertainty lies in parameter identification. For PFF, the crack-surface density and the directional fracture-toughness tensor govern both the onset and the path. Reported calibrations leverage uniaxial and mixed-mode coupon data aligned with material axes, then validate on off-axis panels; the adaptive schemes that concentrate mesh and control residual stiffness show the best energy balance when the crack rotates relative to the fibers [3,10]. For PD, the horizon size, influence function, and, where used, the tensor-involved micromodulus define an intrinsic length scale; convergence and the suppression of zero-energy modes improve when the elastic tensor enters the kernel explicitly, with recent formulations recovering path-independent SIFs through a nonlocal interaction-integral even in strongly orthotropic plates [7–9]. Enrichment-based discretizations are relatively insensitive to mesh rotation once anisotropic eigenfunctions enter the basis, but accuracy still depends on pairing with domain/interaction integrals that honor orthotropy; the same holds for X-IGA, where smoothness reduces, but does not eliminate, the need for careful auxiliary fields (see Table 2) [1,5,6].

Table 2 – Method-specific uncertainty drivers and mitigations in orthotropic crack-tip modeling [1–10]

Method	Primary uncertainty driver	Mitigation reported in the literature	Example evidence
PFF	Identification of Gc(θ) and anisotropic crack-surface density; residual stiffness under turning	Axis-aligned calibration + adaptive refinement; split-energy formulations consistent with orthotropy	Off-axis kinking captured with adaptive PFF and aligned degradation
PD (OSPD/Ti-PD)	Horizon selection; zero-energy modes; kernel anisotropy fidelity	Nonlocal interaction-integral; tensor-involved micromodulus; stability analyses	Path-independent SIFs and orientation-dependent kinks in orthotropic plates
XFEM	Enrichment mismatch to true eigenfields; mode partition error	Use anisotropic tip functions; domain/interaction integrals tuned to orthotropy	Accurate SIFs in composites with nonconforming meshes
X-IGA	Basis selection vs. curvature; auxiliary-field quality for K/T	Smooth CAD-exact geometry; enrichment + interaction integrals	Lower DOFs with comparable K/T accuracy on curved notches
Stroh + Jk	Application to evolving paths; interface angle dependence	Use as calibration/validation targets; embed as auxiliary fields in domain integrals	Closed-form validation for mixed-mode and interfaces

Design-oriented criteria based on the minimum strain-energy density (MSED) supply a bridge between computed crack-tip parameters and engineering decision-making in orthotropic composites. By mapping fiber angle and mode mix to failure envelopes through MSED, the criterion yields direction-dependent extension angles without re-solving the whole boundary-value problem; this favors fast screening of layups and ply orientations once K, T, and mode ratios are available from any of the solvers above [2]. In settings with interfaces, combining MSED-type envelopes with Jk-based mode mix provides a consistent route to predict deflection versus penetration at material boundaries where orthotropy contrasts with isotropy [2,6].

For reproducibility and cross-validation, the literature points to paired studies where PFF-predicted paths or PD-predicted branch patterns are subsequently checked against SIF/T-stress from X-IGA or enriched FEM on snapshots of the emerging crack. Reports that close this loop document improved agreement in kink angles and load levels for instability when T-stress is included, especially in off-axis cracks where the oscillatory character of the near-tip field shifts with material rotation [1,3,5,7–9]. The same studies emphasize that benchmark analytics (Stroh eigensolutions and interface Jk integrals) should remain in the loop to ensure that numerical stability does not mask physical anisotropy in the extracted fracture parameters.

Conclusion

The review consolidates an operational pathway for crack-tip analysis in orthotropic composites. Stroh-type eigensolutions define reference kinematics and supply auxiliary fields for domain and interaction integrals, ensuring reproducible extraction of K, mode mixity, and T-stress. Interaction-integral formulations adapted to orthotropy and orthotropic/isotropic interfaces stabilize mode partition and provide benchmarks for finite-domain calculations.

When crack paths are not prescribed or rotate with fiber angle, phase-field models with anisotropic crack-surface densities and calibrated length scales deliver mesh-objective evolution. For CAD-exact geometries and efficient parametric sweeps, X-IGA and XFEM with anisotropic enrichments recover reference-grade K/T without mesh conformity. Peridynamics complements these tools where finite interaction ranges and fragmentation dominate, with orthotropy-consistent kernels supporting path-independent SIF evaluation. The resulting workflow addresses the stated tasks: it systematizes analytical baselines, differentiates numerical readiness by target outcome, and specifies a validation scheme that couples benchmark analytics with evolution solvers and design-oriented criteria.

For aerospace composite structures, including fuselage skins, wing panels, fairing stiffened panels, and bonded joints, these tools are especially valuable, as they support durability predictions, damage-tolerance assessments, and certification-driven design workflows. The resulting workflow is directly aligned with industry needs at companies like Boeing, Airbus, Bombardier, where multi-scale fracture modeling is essential for safety-critical structural components.

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