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Systems Of Differential Square Equation Of Volter's Model On Simplex

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ABSTRACT

Continuous model of Lotka-Volter's on simplex Sm-1 is studied in the article. There was determined a relationship between tournament and the system of differential square equation, that describes evolution of genetic systems in Lotka-Volter's model.

KEYWORDS

System of differential square equation, alternate matrix, transversality, spurs.

INTRODUCTION

System of differential equation model

$$\dot{x}_k(t) = x_k(t)\left(\sum_{i=1}^{m} a_k x_i(t)\right),\ k=\overline{1,m} \qquad (1)$$

is named differential equation of Volter's model, where $|a_{ki}| \le 1$ and $a_{ki} = -a_{ik}$, but $x_i(t)$ – unknown functions.

Let $A_m = (a_{ki})_{k,i=1}^{m}$, $|a_{ki}| \le 1$, $\forall\, k,i=\overline{1,m}$ alternate matrix, i.e. $A_m^T = -A_m$.

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Definition. Alternate matrix A_m is called *transversal*, if all main minors of even order differs from zero.

Obviously that if A_m – transversal, then $a_{ki} \neq 0$ on different k and i.

Hereinafter we shall consider only transversal alternate matrix, not mentionning word "transversality".

To each alternate matrix

$$A_m = (a_{ki})_{k,i=1}^{m}$$

We shall match the tournament T_m like, that there is corresponding matrix of adjacency to it $signA_m = \left(sign(a_{ki})\right)_{k,i=1}^{m}$ i.e. tops k and i connected with arrow coming from i and k, if $a_{ki} > 0$.

Since A_m –transversal alternate matrix, Definition is considered to be correct.

Definition. Alternate matrix is identified as *transitive (strong)*, if corresponding tournament is *transitive strong* as well. (see No.1).

Alternate matrixes $A_m^{(1)}$ and $A_m^{(2)}$ are identified isomorphic, if corresponding tournaments $T_m^{(1)}$ и $T_m^{(2)}$ are isomorphic as well.

THE MAIN FINDINGS AND RESULTS

The offer 1 [2]. The tournament which corresponds to alternate matrix A is not strong then and only then, when it accurate to isomorphism has the following model:

$$\begin{pmatrix} A_r & A_{r\times s}^{+} \\ A_{s\times r}^{-} & A_s \end{pmatrix}$$

where A_r and A_s alternate matrix, all the elements of the matrix $A_{r\times s}^{+}$ positive, $-\left(A_{r\times s}^{+}\right)^T$, moreover $r + s = m$.

We shall look through the problem of Koshi for the equation (1) with initial condition

$$x(t_0) = \left(x_1(t_0), x_2(t_0), \ldots, x_m(t_0)\right) \in S^{m-1}$$
(2)

Since simplex S^{m-1} compacted, then decision of the problem of Koshi exists and single in gap $(-\infty; +\infty)$. In order to demonstrate that

$x(t) \in S^{m-1}$ when $t \in (-\infty; +\infty)$

where $x(t) = \left(x_1(t), x_2(t), \ldots, x_m(t)\right)$ solution of the problem of Koshi, we shall look through the following function $f : R^m \to R$, for any $x = (x_1, x_2, \ldots, x_m) \in R^m$

$$f(x) = \sum_{k=1}^{m} x_k . \qquad (3)$$

This function is named *spur*.

By means of this function it is possible to study several characteristics of solution of differential equation (1).

(***i***). Let $x(t)$ the solution of differential equation (1). Then the following will be fair:

$$f(\dot{x}(t)) = 0$$

In the reality,

$$f(\dot{x}(t)) = \sum_{k=1}^{m}\left(x_k(t)\left(\sum_{i=1}^{m} a_{ki}x_i(t)\right)\right) = \sum_{k=1}^{m}\sum_{i=1}^{m} a_{ki}x_j(t)x_i(t) = 0$$

So as $a_{ij} = -a_{ji}$.

(***ii***). Let $x(t)$ the solution of differential equation (1) with initial condition (2), then

$f(x(t)) = 1$ for any $t \in (-\infty; +\infty)$.

In the reality, according to (***i***), we'll get

$$\dot{f}(x(t)) = \sum_{k=1}^{m} \dot{x}_k(t) = f(\dot{x}(t)) = 0.$$

Fromhere

$f(x(t)) = f(x(t_0)) = \sum_{k=1}^{m} x_k(t_0) = 1$ for any $t \in (-\infty; +\infty)$.

(***iii***). Let $x(t)$ the solution of differential equation (1) with initial condition

$$x(t_0) = (0, x_2(t_0), \ldots, x_m(t_0)). \quad (4)$$

Then $x(t) = (0, x_2(t), \ldots, x_m(t))$ for any $t \in (-\infty; +\infty)$.

In the reality, we'll look through the solution of the problem of Koshi only comparatively to unknown function $x_1(t)$, considering $x_i(t)$ – given functions, where $i = \overline{2,m}$:

$$\dot{x}_1(t) = x_1(t)\left(\sum_{i=1}^{m} a_{1i}x_i(t)\right), \quad (5)$$

with initial conditions

$$x_1(t_0) = 0 \quad (6)$$

Since the right part of differential equation (5) satisfies the condition of Lipchitz, then $x_1(t)$ the solution of the problem of Koshi for differential equation (5) with initial condition (6) exists and single, i.e. $x_1(t) = 0$ for any $t \in (-\infty; +\infty)$.

Effect 1. Let $x(t)$ the solution of differential equation (1) with initial condition (2), then $x(t) = (x_1(t), x_2(t), \ldots, x_m(t)) \in S^{m-1}$ for any $t \in (-\infty; +\infty)$.

Effect 2. Let $x(t)$ the solution of differential equation (1) with initial condition $x(t_0) = (x_1(t_0), x_2(t_0), \ldots, x_m(t_0)) \in \operatorname{int} S^{m-1}$. (7)

Then $x(t) = (x_1(t), x_2(t), \ldots, x_m(t)) \in \operatorname{int} S^{m-1}$ for any $t \in (-\infty; +\infty)$.

Let the matrix be $A_m = (a_{ki})_{k,i=1}^{m}$ is not strong and let T_m – be a tournament, corresponding to a matrix A_m.

According to the Offer No. 1 we consider, that the matrix A_m has

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$$\begin{pmatrix} A_r & A^+_{r\times s} \\ A^-_{s\times r} & A_s \end{pmatrix}$$

where A_r and A_s – alternate matrixes, all elements of the matrix $A^+_{r\times s}$ positive and $A^-_{r\times s} = -\left(A^+_{r\times s}\right)^T$, here $r + s = m$. Besides, let's imagine, that T_r – sub tournament and T_s –sub tournament T_m –tournament are considered the tournaments according to alternate matrixes A_r and A_s.

Theorem 1. Let $x(t)$ the solution of differential equation (1) with initial condition (7). Then for the free top i of the tournament T_s the equation is executed as following $\lim\limits_{t\to-\infty} x_i(t) = 0$ and for the free top j of the tournament T_r is executed as following $\lim\limits_{t\to+\infty} x_j(t) = 0$.

Proof. According to the effect No. 2, we have $x(t) \in \operatorname{int} S^{m-1}$ for any $t \in (-\infty; +\infty)$. Since the matrixes A_r и A_s – alternate, then majority solutions of inequality $A_r x \le 0$ и $A_s x \ge 0$, accordingly in the simplexes S^{r-1} and S^{s-1}, is not empty [3].

(*i*). Let's consider, that the point $q = (q_1, q_2, \ldots, q_r) \in S^{r-1}$ is considered as a solution of inequity $A_r x \prec 0$. Then the point $\tilde{q} = (q_1, q_2, \ldots, q_r, \underbrace{0,0,\ldots,0}_{s})$ belongs to the simplex S^{m-1}, here $r + s = m$.

It is clear, that the point $\tilde{q} \in S^{m-1}$ is considered to be the solution of inequity for $A_m x \le 0$ besides

$$A_m \tilde{q} = (a_1, a_2, \ldots, a_r, a_{r+1}, \ldots, a_m),$$

where $a_i \le 0$ for any $i = \overline{1,r}$ and $a_{r+i} < 0$ for any $i = \overline{1,s}$, here $r + s = m$.

from the equation (1) we have the following

$$\dot{\ln}(x_k(t)) = \sum_{i=1}^{m} a_{ki} x_i(t) = \left(A_m x(t)\right)_k,$$

$$k = \overline{1,m} \quad (8)$$

Let's enter the indication

$$Ln(t) = \left(\ln(x_1(t)), \ln(x_2(t)), \ldots, \ln(x_1(t))\right)$$

.

We'll look through interproduct vectors $\dot{Ln}(t)$ и $\tilde{q}$:

$$\left(\dot{Ln}(t), \tilde{q}\right) = \left(A_m x(t), \tilde{q}\right) = \left(x(t), A_m^T \tilde{q}\right) = -\left(x(t), A_m \tilde{q}\right) = -\sum_{i=1}^{m} a_i x_i(t) =$$
$$= -\left(a_1 x_1(t) + a_2 x_2(t) + \ldots + a_m x_m(t)\right)$$

Since $x(t) \in \operatorname{int} S^{m-1}$ for any $t \in (-\infty; +\infty)$ and $a_i \le 0$ for any $i = \overline{1,r}$ and $a_{r+i} < 0$ for any $i = \overline{1,s}$, то

$$-\sum_{i=1}^{m} a_i x_i(t) > 0$$

for any $t \in (-\infty; +\infty)$.

From other side, interproduct vectors $\dot{Ln}(t)$ and $\tilde{q}$:

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$$\left(\dot{Ln}(x(t)),\tilde{q}\right)=q_1\,\dot{\ln}\left(x_1(t)\right)+q_2\,\dot{\ln}\left(x_2(t)\right)+\ldots+q_r\,\dot{\ln}\left(x_r(t)\right)=\dot{\ln}\left(\prod_{i=1}^{r}x_i^{q_i}(t)\right)$$

This means, the following equation is executed

$$\dot{\ln}\left(\prod_{i=1}^{r}x_i^{q_i}(t)\right)=-\sum_{i=1}^{m}a_i x_i(t). \qquad (9)$$

Thereby, the function $\varphi(t)=\prod_{i=1}^{r}x_i^{q_i}(t)$ increases and $0<\varphi(t)<1$ for any $t\in(-\infty;+\infty)$. That is why it exists as following

$$\lim_{t\to+\infty}\varphi(t)=\lim_{t\to+\infty}\prod_{i=1}^{r}x_i^{q_i}(t)=C.$$

Then it is clear that

$$\omega(x_0)\subset\left\{\prod_{i=1}^{r}x_i^{q_i}=C\right\},$$

where $\omega(x_0)$ set of limited points of paths.

Let's take the free point $M\in\omega(x_0)$. We'll look through the path

$$M(t)=\left(M_1(t),M_2(t),\ldots,M_m(t)\right)\subset\omega(x_0)$$

points M. Then from (9) we have

$$0=\dot{\ln}\left(\prod_{i=1}^{r}M_i^{q_i}(t)\right)=-\sum_{i=1}^{m}a_i M_i(t)=-(a_1M_1(t)+a_2M_2(t)+\ldots+a_mM_m(t))$$

Since $a_{r+i}<0$ fro any $i=\overline{1,s}$, for the fair of the last equation must be executed as following

$M_{r+1}(t)=M_{r+2}(t)=\ldots=M_m(t)=0$ for any $t\in(-\infty;+\infty)$.

Thereby

$$\omega(x_0)\subset\left\{\prod_{i=1}^{r}x_i^{q_i}=C\right\}\cap\left\{x_{r+1}=x_{r+2}=\ldots=x_m=0\right\}$$

As the digits $r+1, r+2,\ldots,m$ are considered to be the tops of the tournament T_s, then for the free top i of the tournament T_s the following is executed:

$$\lim_{t\to+\infty}x_i(t)=0.$$

t is looked through the case similarly, when $p=(p_1,p_2,\ldots,p_s)\in S^{s-1}$ is considered the solution of inequity $A_s p\geq 0$. In this case we get

$$\lim_{t\to+\infty}x_j(t)=0.$$

Biological sense of the theorem 1.

CONCLUSION

If the tournament of the evolution is not strong, then all biological models can be divided into two classes thereby that models from first class during the process of evolution step by step (monotonously) disappear (collections which are indicated in the theorem 1 through T_s), initial evolution starts in the vicinity of some balanced condition in which are present only the models from first class.

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