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The Continuation Task For Abstract Bicaloric Equation

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ABSTRACT

An ill-posed problem for an abstract bichaloric equation is studied and Tikhonov stability estimate is given.

KEYWORDS

Caloric, abstract, positive, self-adjoint, unbounded, everywhere dense, operator, bicaloric.

INTRODUCTION

Task. You need to find a solution to an abstract bicaloric equation

$$K_+^2 u(t) \equiv \left(\frac{d}{dt} + A\right)^2 u(t) = 0, \; 0 < t < T, \quad (1)$$

satisfying the following conditions:

$$\left.\begin{aligned} u|_{t=l_1} &= u(l_1) \\ u|_{t=l_2} &= u(l_2) \end{aligned}\right\} \qquad (2)$$

where $u(t)$ - abstract function with values in a hilbert space H.

A - constant, positive definite, self-adjoint, linear, unbounded with an everywhere dense domain of definition

$D(A^2)$ (DCH) the operator, acting from H in H, with $u(l_1), u(l_2) \in H$.

MATERIALS AND METHODS

The validity of the representation is proved.

$$u = u_1 + (t - l_1)u_2.$$

The theorem. If u_1 and u_2 if there are solutions to the caloric equation, then the function $u = u_1 + (t - l_1)u_2$ there is a solution to equation (1) and vice versa, for each given abstract bicaloric function there are such functions u_1 and u_2 what

$$u = u_1 + (t - l_1)u_2$$

Proof. 1) If u_1 and u_2 are solutions to the caloric equation, then there is a solution to the bicaloric equation

$$K_+u = K_+\left[u_1 + (t-1_1)u_2\right] = K_+u_1 + u_2 + (t-1_1)\frac{du_2}{dt} + A(t-1_1)u_2 =$$
$$= u_2 + (t-1_1)\left(\frac{du_2}{dt} + Au_2\right) = u_2 + (t-1_1)\cdot K_+u_2 = u_2.$$

So, as

$\frac{du_2}{dt} + Au_2 = 0,$ to $K_+\left(u_1 + (t-1_1)u_2\right) = u_2$ in $K_+u = u_2$.

Applying the operator again K_+, given, what $K_+u_2 = K_+K_+u = 0;$

2) If u solving the bicaloric equation, then there will be such caloric functions u_1, u_2 what $u=u_1+(t-l_1)u_2$.

To prove this statement, it is enough to establish the possibility of choosing u_2.

Put

$$u_2=K_+u,$$
$$u_1=u-(t-l_1)u_2.$$

It remains to show that

$$K_+\left[u-(t-l_1)u_2\right]=0.$$

In fact:

$$K_+u_1=K_+\left[u-(t-l_1)u_2\right]=K_+u-K_+(t-l_1)u_2=$$
$$=K_+u-u_2-(t-l_1)\cdot\frac{du_2}{dt}-A\cdot(t-l_1)u_2=$$
$$=K_+u-u_2-(t-l_1)\cdot\left(\frac{du_2}{dt}-Au_2\right)=K_+u-u_2=0,$$

from here

$K_+u_1=0,\ \ K_+u_2=0.$

The theorem is fully proved.

Using the view

$$u=u_1+(t-l_1)u_2 \qquad (3)$$

The solution of the problem (1) – (2) can be reduced to the solution of the following problems:

$$\begin{cases} K_+u_1=0, & (4)\\ u_1\big|_{t=l_1}=u(l_1) & (5)\end{cases}$$

and

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$$\begin{cases} K_+ u_2 = 0, & (6) \\ u_2\big|_{t=l_2} = u_2(l_2) & (7) \end{cases}$$

where $u_2(l_2) = \dfrac{u(l_1)}{l_2 - l_1} - \dfrac{u_1(l_2)}{l_2 - l_1}$, $\quad u_1(l_2) = \|u(0)\|^{\frac{l_1 - l_2}{l_1}} \|u(l_1)\|^{\frac{l_2}{l_1}}$

task (4) – (5) $0 < t < l_1$ incorrect in the classical sense, a $l_1 < t < T$ correct. Task (4) – (5) we will investigate for conditional correctness by Tikhonov [1]

The theorem. For any solution of problem (4) – (5), the inequality is valid.

$$\|u_1(t)\| \le \|u(0)\| \frac{l_1 - t}{l_1} \cdot \|u(l_1)\|^{\frac{t}{l_1}}.$$

Proof. Consider the function [1]

$$\varphi(t) = \|u_1(t)\|^2 = (u_1, u_2).$$

Differentiating it, we get

$$\varphi'(t) = 2(u_1', u_1) = 2(Au_1, u_1)$$

$$\varphi''(t) = 2(u_1', u_1) + 2(u_1, u_1'') = 2(Au_1, Au_1) + 2(u_1, A^2 u_1).$$

Since the operator is self-adjoint $(m.e. A = A^*)$, to $(u_1, A^2 u_1) = (Au_1, Au_1)$ and, so, $\varphi''(t) = 4(Au_1, Au_1)$.

Now consider the function

$$\psi(t) = ln\varphi(t)$$

Differentiating it, we have

$$\psi''(t) = \frac{1}{\varphi^2(t)}\left[\varphi''(t) \cdot \varphi'^2(t)\right] = \frac{4}{\varphi^2(t)}\left[(Au_1, Au_1)(u_1, u_1) - (Au_1, u_1)^2\right] \ge 0$$

(8)

By virtue of the well-known Bunyakovsky inequality, inequality (8) means that the function $\psi(t)$ it is inverted by concavity upwards, from which it follows that the function $\psi(t)$ on the segment $[0,l_1]$ does not exceed a linear function that takes the same values at the ends of the segment as $\psi(t)$. From (8) follows

$$\psi(t)\le\frac{l_1-t}{l_1}\psi(0)+\frac{t}{l_1}\psi(l_1) \qquad (9)$$

By potentiating the inequality (9), we obtain

$$\varphi(t)\le[\varphi(0)]\frac{l_1-t}{l_1}\cdot[\varphi(l_1)]^{\frac{t}{l_1}},$$

Where from $\|u_1(t)\|\le\|u(0)\|^{\frac{l_1-t}{l_1}}\cdot\|u(l_1)\|^{\frac{t}{l_1}}$

RESULT AND DISCUSSION

Task (6) – (7) $0<t<l_2$ incorrect, a $l_2<t<T$ in the classical sense, it is correct, in the same way as the problem (4) – (5), it can be investigated for conditional correctness by Tikhonov [1]

We prove a theorem that characterizes the stability estimation of the solution of the problem (1) – (2)

The theorem. For any solution of the problem (1) – (2), the inequality is valid

$$\|u(t)\|_H\le\|u(0)\|^{\frac{l_1-t}{l_1}}\|u(l_1)\|^{\frac{t}{l_1}}+$$
$$+(t-l_1)\begin{cases}\dfrac{1}{l_2-l_1}\left(\|u(l_2)\|+\|u(0)\|^{\frac{l_1-l_2}{l_1}}\|u(l_1)\|^{\frac{l_2}{l_1}}\right)^{\frac{t}{l_2}}\cdot\|u(l_1)\|^{\frac{t-l_1}{l_1}}, & l_1<t<l_2\\ \dfrac{1}{T-l_1}\left(\|u(T)\|+\|u(0)\|^{\frac{l_1-T}{l_1}}\|u(l_1)\|^{\frac{T}{l_1}}\right)^{\frac{T-t}{T}}\cdot\|u(l_2)\|^{\frac{t}{l_2}}, & l_2\le t\le T\end{cases} \qquad (10)$$

Note that the inequality (10) implies the uniqueness of the solution of the problem (1) – (2) and the conditional correctness of this problem in the class

$$\{u : \|u(0)\| \leq M\}$$

This theorem is proved by the logarithmic convexity method [1]

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