The American Journal of Engineering and Technology
(ISSN – 2689-0984)
Published: February 26, 2021 | **Pages:** 75-82
Doi: https://doi.org/10.37547/tajet/Volume03Issue02-11

IMPACT FACTOR
2021: 5. 705
OCLC - 1121105677

Application Of Variational Grid Method For The Solution Of The Problem On Determining Mosture Content Of Raw Cotton In A Drum Dryer

Alisher Mamatov
Doctor Of Technical Sciences, Professor, Department Of Mathematics And Informatics, Tashkent Institute Of Textile And Light Industry, Uzbekistan

Ravshanbek Zulunov
Candidate Of Science (PhD) In Physics And Mathematics, Head Of The Department Of Software Engineering, Ferghana Branch Of Tashkent University Of Information Technologies, Uzbekistan

Munira Sodikova
Assistant, Ferghana Branch Of Tashkent University Of Information Technologies, Uzbekistan

ABSTRACT

In the article a one-boundary parabolic problem on determining moisture of raw cotton in a drum dryer is solved. Numerical results of the considered problem are taken on the method of Bubnov-Galerkin, they are compared with the experimental data. It is shown that the suggested mathematical model and its numerical algorithm adequately describe the process of drying of raw cotton. The stability of the approximate solution is set.

KEYWORDS

Mathematical model, algorithm, drum dryer, raw cotton.

INTRODUCTION

The product quality largely depends on preparing cotton for processing at cotton factories, and primarily, on the process of drying the raw cotton. The main task in drying cotton is to bring its humidity to the standardized. However, this task is not always solved successfully with modern technology of raw cotton processing. To intensify the drying process heat transfer fluids are used, which lead to worsening of the qualitative indices of the raw material.

The main disadvantage of the existing drying technology of which lead to the quality impairment of fiber in the pre-processing is uneven drying, overheating, overdrying of the fiber. The fiber becomes brittle and crisp, its structural and mechanical properties deteriorate. Therefore, theoretical studies of thermal and humidity state of the raw cotton during its drying in drum dryers play a crucial role in the investigation. [1-5]

This paper discusses the mathematical model for determining the temperature and humidity of fields of components of raw cotton in the drying process.

PROBLEM STATEMENT AND METHOD OF SOLUTION

Let us consider the problem of drying raw cotton in a direct-flow drum dryer. Suppose that convective heat transfer occurs according to Newton's law between raw cotton and air. Then, for determining the temperature and humidity of the raw cotton in drying we can compose an initial-boundary value problem in the parabolic type in the form [6-9]:

$$\begin{cases} \dfrac{\partial T}{\partial \tau} = a\dfrac{\partial^2 T}{\partial x^2} - v\dfrac{\partial T}{\partial x} - \alpha_{11}(T - T_B) + \alpha_{12}\dfrac{\partial U}{\partial \tau} \\ \dfrac{\partial U}{\partial \tau} = a_m\dfrac{\partial^2 U}{\partial x^2} + a_m\delta\dfrac{\partial^2 T}{\partial x^2} - v\dfrac{\partial U}{\partial x} \end{cases} \qquad (1)$$

with initial

$T(x,0)=T_0(x)$, $U(x,0)=U_0(x)$(2)

and boundary conditions

$$\left.\frac{\partial U}{\partial x}\right|_{x=0} = 0 \quad , \quad \left.\frac{\partial U}{\partial x}\right|_{x=l} = 0$$

$$-\lambda\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0, \qquad \lambda\left.\frac{\partial T}{\partial x}\right|_{x=l} = \alpha_1(T - T_c) \qquad (3)$$

where $a = \dfrac{\lambda}{c\rho}$, $a_m = \dfrac{\lambda_m}{c_m\rho}$, $\alpha_{11} = \dfrac{\alpha}{c\rho}$, $\alpha_{12} = \dfrac{\varepsilon r_{21}}{c}$,

T, T_B,, T_c are respectively the temperature of the raw cotton dried by an agent (air) and external environment; U is the moisture content of raw cotton and air; c, λ, ρ, v are respectively heat capacity, heat conductivity, density and speed of the motion of the raw cotton; α ,α_1 are volumetric superficial coefficient of heat transfer between the raw cotton and air; ε is the coefficient of phase transition, r_{21} is heat of vaporization, τ is the time of drying, λ is the length of the drum.

Note that initial and boundary conditions with variable coefficients are considered.

Published: February 26, 2021 | Pages: 75-82
Doi: https://doi.org/10.37547/tajet/Volume03Issue02-11
IMPACT FACTOR
2021: 5. 705
OCLC - 1121105677

To solve this problem, we use the Galerkin method. Let's introduce two sets of basis functions and denote them by $\{\varphi_i\}$, $\{\psi_i\}$. We require elements of basis functions to possess the second derivative on spatial variables.

We will search for approximate solutions of the system in the form of [10]

$$T=\sum_{k=1}^{N} c_k(\tau)\cdot\varphi_k(x); \quad U=\sum_{ki=1}^{N} d_k(\tau)\cdot\psi_k(x) \tag{5}$$

where coefficients of $c_k(\tau)$, $d_k(\tau)$ are determined from the system

$$\int_0^l \frac{\partial T}{\partial\tau}\cdot\varphi_i(x)dx = a\int_0^l \frac{\partial^2 T}{\partial x^2}\cdot\varphi_i(x)dx - \vartheta\int_0^l \frac{\partial T}{\partial x}\cdot\varphi_i(x)dx - \\ -\alpha_{11}\int_0^l (T-T_B)\cdot\varphi_i(x)dx + \alpha_{12}\int_0^l \frac{\partial U}{\partial\tau}\cdot\varphi_i(x)dx$$

$$\int_0^l \frac{\partial U}{\partial\tau}\cdot\psi_i(x)dx = a_m\int_0^l \frac{\partial^2 U}{\partial x^2}\cdot\psi_i(x)dx + a_m\delta\int_0^l \frac{\partial^2 T}{\partial x^2}\cdot\psi_i(x)dx - \\ -\vartheta\int_0^l \frac{\partial U}{\partial x}\cdot\psi_i(x)dx \tag{6}$$

Using the formula of integration by parts and taking into account boundary conditions, we obtain the following system of ordinary differential equations:

$$\begin{cases} \sum_{k=1}^{N}\alpha_{ik}c'_k + \sum_{k=1}^{N}\beta_{ik}c_k + \sum_{k=1}^{N}\gamma_{ik}d_k = \bar{f}_{1i} \\ \sum_{k=1}^{N}\bar{\alpha}_{ik}d'_k + \sum_{k=1}^{N}\bar{\beta}_{ik}c_k + \sum_{k=1}^{N}\bar{\gamma}_{ik}d_k = \bar{f}_{2i} \end{cases} \tag{7}$$

with initial conditions

$$\begin{cases} \sum_{k=1}^{N}\alpha_{ik}c_k(0) = \int_0^{\lambda} T_0(x)\,\varphi_k(x)dx \\ \sum_{k=1}^{N}\bar{\alpha}_{ik}d_k(0) = \int_0^{\lambda} U_0(x)\,\psi_k(x)dx \end{cases} \tag{8}$$

Published: February 26, 2021 | **Pages:** 75-82
Doi: https://doi.org/10.37547/tajet/Volume03Issue02-11

where $\alpha_{ik} = \int_0^{\lambda} \varphi_i(x)\varphi_k(x)dx$, $\overline{\alpha}_{ik} = \int_0^{\lambda} \psi_i(x)\,\psi_k(x)dx$

We write the systems (8) and (9) in a vector form:

$$\begin{cases} Q_n \cdot \dfrac{dC_n(\tau)}{d\tau} + P_n C_n(\tau) + G_n D_n(\tau) = F_{1n}(\tau) \\ \tilde{Q}_n \cdot \dfrac{dD_n(\tau)}{d\tau} + \tilde{P}_n D_n(\tau) + \tilde{G}_n C_n(\tau) = F_{2n}(\tau) \\ Q_n C_n(0) = F_{10} \\ \tilde{Q}_n D_n(0) = F_{20} \end{cases} \qquad (9)$$

where $Q_n=(\alpha_{ik})$, $P_n=(\beta_{ik})$, $G_n=(\gamma_{ik})$, $\tilde{Q}_n = (\tilde{\alpha}_{ik})$ $\tilde{P}_n = (\tilde{\beta}_{ik})$ and $\tilde{G}_n = (\tilde{\gamma}_{ik})$ are square matrices sized (NxN); $C_n(\tau)=(c_1(\tau),c_2(\tau),\dots,c_n(\tau))^T$, $D_n(\tau)=(d_1(\tau),\ d_2(\tau)\ ,\dots,d_n(\tau))^T$ are required vectors; $F_{1n}(\tau)=(f_{11}(\tau),f_{12}(\tau),\dots,f_{1n}(\tau))^T$, $F_{2n}(\tau)=(f_{21}(\tau),f_{22}(\tau),\dots,f_{2n}(\tau))^T$ are given vectors; the elements of the vectors $F_{10}(\tau)=(f_{01}(\tau),f_{02}(\tau),\dots,f_{0n}(\tau))^T$ и $F_{20}(\tau) = \left(\tilde{f}_{01}(\tau),\ \tilde{f}_{02}(\tau),\ \dots,\ \tilde{f}_{0n}(\tau)\right)^T$ are determined by the right part of the system (8).

As is known, among theories of ordinary differential equations, in non-degeneracy and positive certanty of the matrix, composed from coefficients of the system, the system (9) has a unique solution.

Let's explore the question of the stability of the problem (9). Suppose that coordinate systems $\{\varphi_i(x)\}$,$\{\psi_i\}$ are strongly minimal in the space $L_2(\Omega)$ i.e. there exists such a constant independent from n, that $0 < q < q_i^n$, where q_i^n are eigenvalues of the matrix

$$Q_n = \{(\varphi_k, \varphi_j)L_2\}_{k,\ j=1}^{n}\ \tilde{Q}_n = (\tilde{\alpha}_{ik}).$$

Let's say instead of the Galerkin system (9), we are solving a "disturbed" problem:

$$\begin{cases} (Q_n + \Gamma_n) \cdot \dfrac{d\tilde{C}_n(\tau)}{d\tau} + (P_n + \Gamma^1{}_n)\tilde{C}_n(\tau) + (G_n + \Gamma^2{}_n)\tilde{D}_n(\tau) = F_{1n}(\tau) + \delta^1{}_n \\ (\tilde{Q}_n + \tilde{\Gamma}_n) \cdot \dfrac{d\tilde{D}_n(\tau)}{d\tau} + (\tilde{P}_n + \tilde{\Gamma}^1{}_n)\tilde{D}_n(\tau) + (\tilde{G}_n + \tilde{\Gamma}^2{}_n)\tilde{C}_n(\tau) = F_{2n}(\tau) + \delta^2{}_n \\ (Q_n + \Gamma_n)\tilde{C}_n(0) = F_{10} + \varepsilon_{10} \\ (\tilde{Q}_n + \tilde{\Gamma}_n)\tilde{D}_n(0) = F_{20} + \varepsilon_{20} \end{cases} \qquad (10)$$

Where $\tilde{C}_n(\tau)$, $\tilde{D}_n(\tau)$ is the solution of the disturbed problem.

The Galerkin process for this problem is called stable, if there exist an independent from the *n* positive constant p_i, that for sufficiently few norms of matrices $\|\Gamma_n^0\|$, $\|\Gamma_n\|$, $\|\Gamma'_n\|$ and norms of vectors $\|\varepsilon_0\|$, $\|\varepsilon_n\|$ inequality is exercised

$$\left\|\tilde{C}_n(\tau)-C_n(\tau)\right\|_{E_n} \le p_0\|\varepsilon_0\|+p_1\|\varepsilon_n\|+p_2\|\Gamma_n^0\|+p_3\|\Gamma_n\|+p_4\|\Gamma'_n\| \qquad (9)$$

$$\left\|\tilde{d}_n(\tau)-d_n(\tau)\right\|_{E_n} \le \tilde{p}_0\|\tilde{\varepsilon}_0\|+\tilde{p}_1\|\tilde{\varepsilon}_n\|+\tilde{p}_2\|\tilde{\Gamma}_n^0\|+\tilde{p}_3\|\tilde{\Gamma}_n\|+\tilde{p}_4\|\Gamma'_n\|$$

An approximate solution $U(r,\tau)=(\theta_{1n},\theta_{2n}(r,\tau))^T$ is called stable in the space $L_2(\Omega)$, if it has a place of inequality, analogical (9), for a difference $\left\|\tilde{U}(r,\tau)-U(r,\tau)\right\|$,

where $\tilde{U}(r,\tau)=\left(\tilde{\theta}_{1n}(r,\tau),\tilde{\theta}_{2n}(r,\tau)\right)^T$, $\quad \tilde{\theta}_{in}(r,\tau)=\sum_{k=1}^{n}\tilde{a}_{iK}(\tau)\cdot\varphi_K(r)$.

May introduced errors $\Gamma_n, \tilde{\Gamma}_n\ \Gamma^1{}_n\ \tilde{\Gamma}^1{}_n\ \Gamma^2{}_n\ \tilde{\Gamma}^2{}_n$ are such

$$\|\Gamma_n\|\le e_1q,\quad \|\tilde{\Gamma}_n\|\le e_2q,\quad \|\Gamma^1{}_n\|\le e_3q,\quad \|\tilde{\Gamma}^1{}_n\|\le e_4q,\quad \|\Gamma^2{}_n\|\le e_5q,\quad \|\tilde{\Gamma}^2{}_n\|\le e_6q$$

$0\le e_i\le 1,\quad q>0.$ (11)

Denote by $Z_n(\tau)=\tilde{G}_n(\tau)-G_n(\tau)$.

From the system equation (10) we subtract the system of equation (9). We multiply the obtained equation by $\dot{Z}_n(\tau)$ by a scalar, that is

$$\frac{1}{2}\frac{d}{d\tau}\left((Q_n+\Gamma_n)Z_n,\,Z_n\right)+\left((P_n+\Gamma'_n)Z_n,\,\dot{Z}_n\right)=\left(\varepsilon_n,\,\dot{Z}_n\right)+\left(\Phi_n,\,\dot{Z}_n\right) \qquad (12)$$

where $\quad \Phi_n(\tau)=-\Gamma_n\cdot\dot{G}(\tau)-\Gamma'_n\ G_n(\tau)$.

Since the matrix P_n has been positive definite, then

$$\left((P_n+\Gamma'_n)Z_n,\,\dot{Z}_n\right)_n\ge 0.$$

So, estimating members of the right part of the congruence

$$\left|\left(\varepsilon_n,\,\dot{Z}_n\right)_{E_n}\right|\le\frac{1}{2\varepsilon_1}\|\varepsilon_n\|^2+\frac{1}{2}\varepsilon_1\|Z_n\|^2$$

and $\quad \left|\left(\Phi_n(\tau),\,\dot{Z}_n\right)\right|\le\frac{1}{2\varepsilon_1}\|\Phi_n(\tau)\|^2+\frac{1}{2}\varepsilon_1\|Z_n\|^2$

we obtain

$$\frac{1}{2}\frac{d}{d\tau}\left((Q_n+\Gamma_n)Z_n,\,Z_n\right)\le\varepsilon_1\|Z_n\|^2+c_1\left(\|\varepsilon_n\|^2+\|\Phi_n(\tau)\|^2\right)$$

Let's integrate last inequations over τ. Taking into account the inequation $\|Z_n\|^2_{E_n} \le \frac{1}{2}\|\tilde{U}-U\|^2_{L_2}$

we obtain

$$((Q_n+\Gamma_n)Z_n,\, Z_n)_{E_n} \le 2\varepsilon_1\int_0^\tau \|\tilde{U}-U\|^2_{L_2}\, d\tau + c_1\int_0^\tau \left(\|\varepsilon_n\|^2 + \|\Phi_n(\tau)\|^2_{L_2}\right) d\tau + ((Q_n+\Gamma_n)Z_n(0),\, Z_n(0))_{E_n}$$

On the other hand,

$$((Q_n+\Gamma_n)Z_n,\, Z_n)_{E_n} \ge (Q_nZ_n,\, Z_n) - e_1q\|Z_n\|^2_{E_n} \ge (1-e_1)\|\tilde{U}_n - U_n\|^2_{L_2},$$

$$((Q_n+\Gamma_n)Z_n(0),\, Z_n(0))_{E_n} \le (Q_nZ_n(0),\, Z_n(0))_{E_n} + e_1q\|Z_n\|^2_{E_n} \le c_2(1+e_1)\|\tilde{U}(r,\,0)-U(r,\,0)\|^2_{L_2(\Omega)}$$

Then we obtain a differential inequation for $y_n(\tau)$, that is

$$\frac{dy_n(\tau)}{d\tau} \le M\cdot y_n(\tau) + F_n(\tau)$$

from which, in turn, by the power of the theorem on differential inequations this inequation is followed:

$$\frac{dy_n(\tau)}{d\tau} \le e^{G_1\tau}\cdot F(\tau)$$

From it

$$\|\tilde{U}_n(r,\,\tau)-U_n(x,\,\tau)\|^2_2 \le p_0\|\varepsilon_{10}\|^2 + p_1\|\varepsilon_{20}\|^2 + p_2\|\Gamma_n\|^2 + p_3\|\tilde{\Gamma}_n\|^2 + p_4\|\Gamma'_n\|^2 + p_5\|\tilde{\Gamma}^1{}_n\|^2 +$$
$$+ p_6\|\Gamma^2{}_n\|^2 + p_7\|\tilde{\Gamma}^2{}_n\|^2$$

where constant $p_i\,(i=\overline{0,7})$ does not depend on *N*. Consequently,

$$\|\tilde{G}_n(\tau)-G_n(\tau)\|^2_{E_n} \le \frac{1}{q}\|\tilde{U}_n(r,\,\tau)-U_n(r,\,\tau)\|^2_2 \le \frac{1}{q}\omega^2$$

where ω^2 is a right part of the inequation (14). From the last relators we derive the stability of the algorithm constructing an approximate solution and numerical stability of the approximate solution in $L_2(\Omega)$.

For the numerical solution of problem (9), we use the method of differences of schemes with respect to temporal variables. Trying basis functions in this way and building implicit difference schemes on the interval *[0;l]* we obtain a system of algebraic equations.

$$\begin{cases} (Q_n+\Delta\tau P_n)\cdot C_n^{l+1} + G_nD_n^{l+1} = F_{1n}^l - Q_nC_n^l \\ \tilde{G}_nC_n^{l+1} + (Q_n+\Delta\tau\tilde{P}_n)\cdot D_n^{l+1} = F_{2n}^l - Q_nD_n^l \\ Q_nC_n^0 = F_{10} \\ Q_nD_n^0 = F_{20} \qquad l=0,1,2,3,\ldots M \end{cases} \qquad (13)$$

The system of algebraic equations (13) is solved by the Gauss method. The found values $C_n^{\prime}(\tau), D_n^{\prime}(\tau)$ inserting in (5) we find the temperature and moisture content of the raw cotton in the process of drying (Pic.1,2).

PILOT STUDIES AND SNALYSIS OF THE RESULTS

Analysis of the solution of the suggested method for determining the temperature and humidity of the raw cotton were carried out at the following parameter values:

$$\lambda=0{,}09\ W/m\cdot K;\ c=1700\ J/(kg\cdot{}^{0}C);\ \rho=40\ kg/m^{3};\ W_{н}=14{,}3\%,21\%,;$$

$$T_{н}=10^{0};\quad T_{в}=100^{0},140^{0};200^{0};\quad \varepsilon=0{,}8;r_{21}=2082000\ J/kg;\ v=1{,}5m/c$$

$$\alpha=124\ J/(c\ m^{3\,0}C),\ \alpha_1=2{,}5\ J/(c\ m^{2\,0}C)$$

To compare the calculated and experimental data, we will use the results of the pilot research, which was conducted in the drum dryers of the 2SB-10 type. When tested, machine harvest of grade 2, 3 and 4 raw cotton with the 14,3%; 21% reference humidity served as a processing object.

Length of drum dryer	Humidity of raw cotton in W=14,3%, T=200	Humidity of raw cotton in W=14,3%, T=100	Humidity of raw cotton in W=21%, T=140
2	13.97	14.21	20.41
4	12.65	13.95	19.66
6	11.88	13.68	18.83
8	11.35	13.31	18.12
10	10.94 (11.35 ex)	13.02 (13.6)	17.32 (17.67)

Table 1. Changes in the humidity of raw cotton on the length of a drum dryer

Length of drum dryer	Temperature of raw cotton in W=14,3%, T=200	Temperature of raw cotton in W=14,3%, T=100	Temperature of raw cotton in W=21%, T=140
0	10	10	10
2	29,4	17,8	15,8
4	36,7	23,7	21,7
6	47,1	26,9	29,9
8	51,01	31,1	35,1
10	57,4 (56 ex)	38,5 (37 ex)	40,3 (39 ex)

Table 2. Changes in the temperature of raw cotton on the length of a drum dryer

Published: February 26, 2021 | **Pages:** 75-82
Doi: https://doi.org/10.37547/tajet/Volume03Issue02-11

CONCLUSION

The comparison of the experimental data on changes in the humidity and temperature of raw cotton in the drum dryer 2SB-10 and analysis on the approximate solution show that the relative error constitutes no more than 5% (Tables 1,2). This allows to use the given algorithm to calculate the temperature and moisture content of raw cotton during the drying process.

REFERENCES

1. Lykov A.V. (1978). Teplomassoobmen (spravochnik) [Heat and mass transfer (reference)]. Moscow: Energiya [Energy], p.480.
2. Parpiev A.P., Kayumov A.H., Pardayev H. (2016). Effect of temperature of steady heating components of cotton-seed at drying process. *European science review*, 7-8, pp.205-207.
3. Parpiyev A.P., Kayumov A.H., Axmatov N. (2016). Definition of area of soft temperature drying condition.*European science review*, 7-8, pp.208-211.
4. Parpiyev A.P., Kayumov A.H., (2017). Analytical review of the applied temperature in the process of drying raw cotton, *Scientific and Technical Journal of FerPI*, 1, pp.29-34.
5. Lykov A.V. (1978). Heat and mass transfer. Moscow: State Energy Publishing House, p.450.
6. Schekoldin M.N. (1958). Heat moisture state of raw cotton, Moscow: State Gizlegprom Publishing House, p.74.
7. Evans, Lawrence C. (2010) [1998], Partial differential equations (PDF), Graduate Studies in Mathematics, p.19.
8. Providence, R.I. American Mathematical Society, doi:10.1090/gsm/019, ISBN 978-0-8218-4974-3, MR 2597943
9. Hazewinkel, Michiel, ed. (2001) [1994]. Parabolic partial differential equation, numerical methods, *Encyclopedia of Mathematics*, Springer Science+Business Media B.V. / Kluwer Academic Publishers, ISBN 978-1-55608-010-4
10. Mikhlin S.G. (1966). Chislennaya realizatsiya variatsionnykh metodov [Numerical implementation of variational methods], Moscow: Nauka [The science], p.432.
11. M. V. K. Chari and S. J. Salon, Numerical methods in electromagnetism, San Diego : Academic Press, 2000, pp.143-187
12. Ascher U.M., Greif C. (2011). A First Course in Numerical Methods. *SIAM*, Philadelphia, p.580.
13. Karimov U. et al. USING NEW INFORMATION TECHNOLOGIES IN DISTANCE LEARNING SYSTEM //НОВАЯ ПРОМЫШЛЕННАЯ РЕВОЛЮЦИЯ В ЗЕРКАЛЕ СОВРЕМЕННОЙ НАУКИ. – 2018. – С. 9-11.