Mathematical Model of a Controlled Shaft-Type Dryer Under Parametric Uncertainty

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Abstract

This article presents the development of a state-space mathematical model for controlling the wheat drying process in a shaft-type dryer. The drying process is considered a complex multivariable, nonlinear, and distributed-parameter dynamic object characterized by simultaneous heat and mass transfer, transport delay, external disturbances, and variable raw-material properties. Based on the physical characteristics of the process, state, control, output, and disturbance vectors are formulated. Energy balance equations for wheat grains and the drying agent, together with moisture transfer and air humidity equations, are used to construct a nonlinear dynamic model. The nonlinear equations are linearized around a nominal operating point using the first-order Taylor expansion, resulting in a state-space representation. Transport delay associated with grain movement through the dryer is also incorporated into the model. The developed model provides a mathematical basis for analyzing process dynamics and designing advanced control algorithms aimed at improving drying stability, product quality, and energy efficiency.

Keywords: Shaft-type dryer, wheat drying, mathematical model, state-space model, drying process, control system, heat transfer, mass transfer, moisture content, grain temperature, drying agent, transport delay, nonlinear system, linearization, external disturbances, energy balance, moisture transfer, dynamic model, multivariable system, control vector, state vector, output vector, air flow, heat power, drying kinetics, process control, automation, modeling, energy efficiency, grain quality.

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1. Introduction

A shaft-type dryer is a complex multi-input multi-output (MIMO), distributed-parameter, and nonlinear dynamic object. During the drying process, heat and mass transfer occur simultaneously, and the control signals (heat-transfer-agent temperature, air flow rate, product feed rate, etc.) together with external environmental disturbances exert a significant influence on the dynamics of the system. In addition, the technological process is characterized by transport delays, inertia of sensors and actuating mechanisms, and variability of raw-material parameters, all of which must be taken into account in the mathematical model.

The generalized state-space model of the shaft-type dryer is written as follows:

$$\dot{x}(t) = A(\Delta)x(t) + A_d(\Delta)x(t - \tau) + B(\Delta)u(t) + E(\Delta)w(t)$$

$$y(t) = C(\Delta)x(t) + D(\Delta)u(t)$$

where:

$x(t) \in \mathbb{R}^4$ — the state vector;

$u(t) \in \mathbb{R}^3$ — the control (input) signals;

$y(t) \in \mathbb{R}^2$ — the output parameters;

$w(t)$ — the external disturbances;

τ — the transport delay;

Δ — the parametric uncertainty.

The state vector includes the following technological parameters:

$$x(t) = [\ T_g,\ T_a,\ W,\ H\ ]^T$$

where: T_g — grain (product) temperature; T_a — drying-agent (hot air) temperature; W — product moisture content; H — humidity of the air in the drying chamber.

The control (input) vector:

$$u(t) = [\ Q,\ G,\ V_f\ ]^T$$

where: Q — heat power supplied; G — air flow rate; V_f — product feed (throughput) rate.

The output vector:

$$y(t) = [\ W_{out},\ T_{out}\ ]^T$$

where: W_{out} — outlet moisture content; T_{out} — outlet temperature (temperature of the drying agent at the exit of the drying chamber).

Remark. In what follows, the symbols T_g and T_a are used consistently to denote the temperatures of the grain and the drying agent, respectively; the symbol T_m used in the earlier draft version of this chapter has been replaced by T_g, thereby ensuring a uniform notation system throughout the chapter.

Rationale for the choice of state variables. The selection of the four-dimensional state vector $x = [T_g, T_a, W, H]^T$ is not arbitrary but is based on the following two criteria: (1) direct relevance to the control objective — the final quality index of the dissertation (outlet moisture content W_{out} and energy consumption) is expressed directly and exclusively through these four variables (Section 2.3); (2) the decomposition hypothesis substantiated in Section 2.2 — since the drying chamber, the heat generator, and the air-supply system are regarded as weakly coupled subsystems, each of them is adequately described by a single scalar state variable (T_g, T_a, W, and H, respectively). The pressures p_k, p_u (Fig. 2.2) and the distribution along the shaft height were not introduced as separate state variables, because: pressure differences are directly linked to the control signal G through fan performance on a short time scale and can be accounted for as a static (algebraic) relationship rather than as a separate dynamic state; whereas the height-wise distribution genuinely requires a distributed-parameter (PDE) description, but the main dynamic effect of this description is introduced into the lumped, finite-dimensional model in an approximated form through the delay matrix A_d introduced in Section 2.5. Thus, the selected four-dimensional model represents the minimal sufficient description — one that preserves the dynamic properties of the process necessary for control while retaining the lowest possible dimensionality.

Additional remark on the adequacy of the reduction. The validity of the transition from the distributed-parameter (PDE) description to the four-dimensional lumped model relies on a clear separation of time scales: the characteristic time constant of the moisture dynamics is approximately 900 s and that of the thermal dynamics is approximately 600 s (these values are also used in the numerical stability verification of the closed-loop system with an ANFIS controller in Section 3.15.5), whereas the spatial propagation time of the grain layer along the shaft is considerably shorter than these ($\tau \approx$ 15–40 minutes). This more-than-one-order-of-magnitude difference between the two time scales (in the sense of singular perturbation) makes it possible to embed the fast, spatially distributed states in a "damped" (singularly perturbed) form within the delay operator A_d, thereby physically and mathematically justifying the ability of the four-dimensional model to describe the dominant (slow) dynamics without loss of accuracy. A quantitative comparison of the reduction error of the model against a full PDE simulation is regarded as one of the promising directions for future research.

2. Structure and Decomposition of the Technological Process

Based on an analysis of the equipment involved in the drying process and their interdependencies, the process and its equipment may be regarded as a multi-dimensional (in terms of the number of parameters) and multi-connected (in terms of the interdependence of units and their parameters) control object (Fig. 2.1). Such multi-dimensionality and multi-connectedness complicate both the monitoring and control of the drying technological process, and also make the process modeling task more difficult. It is therefore necessary to reduce the multi-dimensionality and multi-connectedness of the process as far as possible, without compromising the completeness with which the process characteristics are represented.

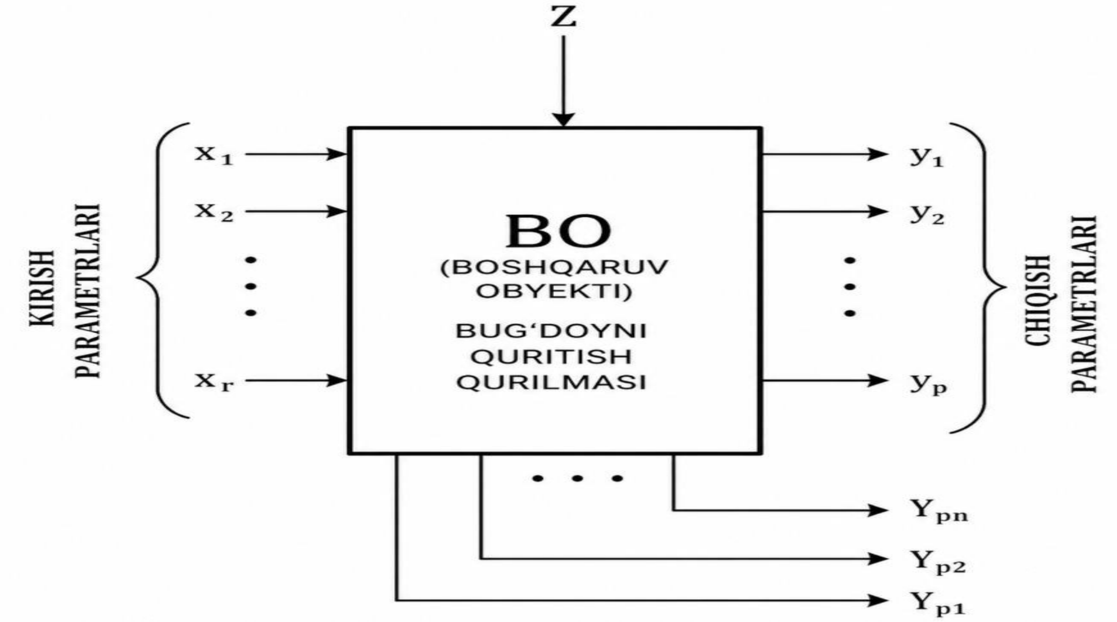

Fig. 1. Scheme representing the parameters of the drying-process control object. xi,r — input parameters (initial moisture content, temperature, and flow rate of the wheat); yi,p — output parameters (moisture content, temperature, and quality of the dried wheat); Z — humidity and influence of the external environment; Ypn — heat and energy consumption of the drying unit.

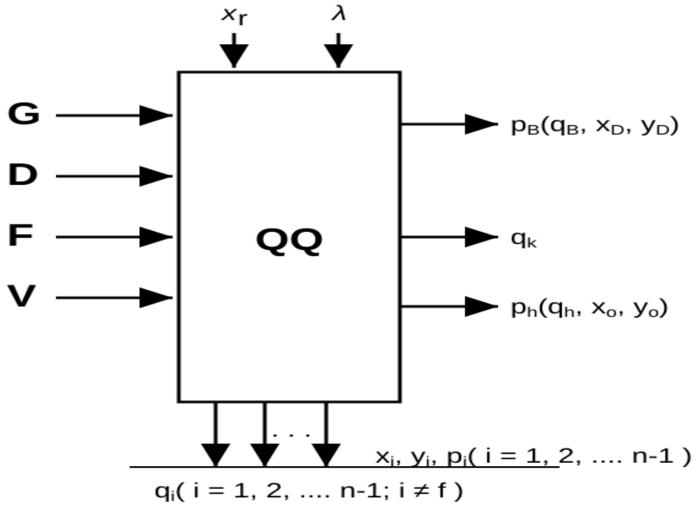

Fig. 2. Simplified scheme representing the parameters of the drying-process control object. QQ — drying chamber; G — flow rate of wet wheat fed to drying; D — flow rate of the drying agent (hot air); F — heat flow supplied to the drying process; V — volume of air delivered by the fan; xi — input parameters; yi — output parameters; Z — control action (drying regime, air temperature, or flow rate); q — temperature or humidity at the control point of the drying chamber; pk, pu — pressures at the lower and upper sections of the drying unit.

For this purpose, certain simplifying assumptions may be adopted. For instance, the moisture content within the wheat grain during drying may be regarded as the primary controlled parameter. In this case, the essential parameters are extracted from the complex multi-parameter system, and quantities such as moisture content, drying-agent temperature, and air-flow velocity are studied to represent the state of the drying process:

$x_i = f(P_i, t_i), \quad t_i = \varphi(x_i)$

One effective method for studying the equipment of the drying unit while reducing the dimensionality and the number of interconnections of its parameters consists of separately analyzing the operation of each element of the equipment complex and its associated parameters. This approach is generally referred to as the **decomposition method**. Taking into account the interconnections between the technological units, the equipment complex used in the wheat-drying process forms a weakly coupled technological system: the product stream leaving the receiving hopper is transferred sequentially to the preliminary cleaner, then to the shaft-type dryer, and finally to the cooling unit (Fig. 2.3). Consequently, in this research it is appropriate to examine the technological process of wheat drying on the basis of the decomposition method.

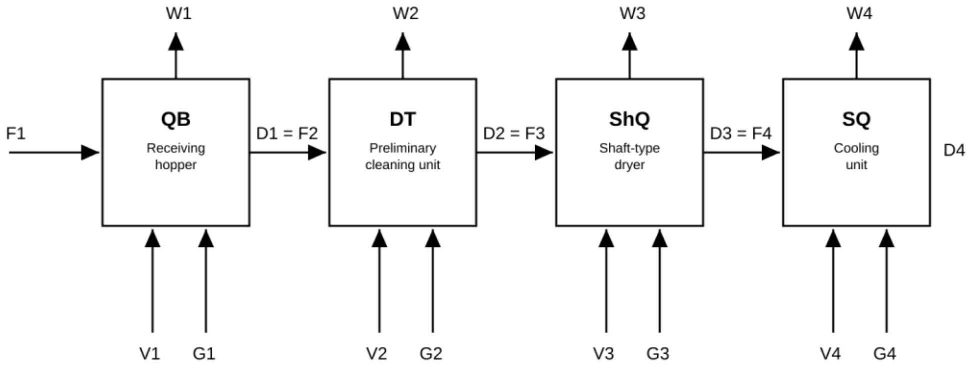

Fig. 2.3. Sequence of connection of the drying-equipment complex forming an open-loop system: QB — receiving hopper; DT — preliminary cleaning unit; ShQ — shaft-type dryer (main object of study); SQ — cooling unit. Fi — the inflow of product into unit i; Di — the outflow of product from unit i (Di = Fi+1); Wi — the loss flow of dust/vapor released in unit i; Vi, Gi — the air flow rate and heat power, respectively, supplied to unit i.

The mathematical model of the drying system under investigation analyzes the interdependencies among the drying chamber, the heat generator, and the air-delivery systems. The main parameters adopted for the drying process are the moisture content of the wheat, the influence of the external environment, the temperature of the drying agent, the air flow rate, and the drying duration. Although a certain degree of feedback exists among these parameters, it is relatively weak, since as the moisture content of the wheat decreases within the drying chamber, the conditions of heat and mass transfer change, which in turn affects the subsequent stages of the drying process. For this reason, these feedback effects may be neglected as a first approximation. In this case, the technological process of wheat drying is regarded as an open-loop system with units sequentially interconnected along the technological flow streams, which makes it possible to formulate the local energy and mass balances presented below (Sections 2.3–2.5) separately and subsequently to combine them into a single state-space model.

Physical Foundations of the Process: Energy and Mass Balances

The equations describing the dynamic behavior of the shaft-type dryer follow from the law of conservation of energy and the mass-balance equations.

Energy balance for the grain. The difference between the amount of heat transferred to the grain and the energy expended on evaporating moisture determines the rate of change of the grain temperature:

$$m_g c_g \, dT_g/dt = hA(T_a - T_g) - r \cdot m_g \, dW/dt$$

where m_g — mass of the wheat (grain); c_g — specific heat capacity; h — heat-transfer coefficient; A — heat-transfer surface area; r — latent heat of vaporization. The first term $hA(T_a - T_g)$ represents the heat transferred to the grain, while the second term $r \cdot m_g \, dW/dt$ represents the energy expended on evaporating the moisture. As a result:

$$dT_g/dt = [hA/(m_g c_g)](T_a - T_g) - (r/c_g) \, dW/dt$$

Energy balance for the drying agent. The hot air receives heat from the generator and transfers it to the grain and to the external environment:

$$m_a c_a \, dT_a/dt = Q - hA(T_a - T_g) - G \cdot c_a(T_a - T_{env})$$

$$dT_a/dt = Q/(m_a c_a) - [hA/(m_a c_a)](T_a - T_g) - (G/m_a)(T_a - T_{env})$$

Moisture exchange. The drying kinetics are described by the Lewis model:

$$dW/dt = -\,k(T_g)(W - W_e)$$

where $k(T_g)$ — the temperature-dependent drying coefficient; W_e — the equilibrium moisture content.

Air humidity. The humidity of the air in the drying chamber is determined from the mass balance:

$$dH/dt = (m_g/m_a)(-dW/dt) - (G/m_a)(H - H_{env})$$

The four equations above form a closed system of nonlinear differential equations in the state variables $\{T_g, T_a, W, H\}$ and constitute the initial (base) point of the linearized state-space model presented in Section 2.5.

Transport Delay and External Disturbances

In shaft-type dryers, a transport delay is present because of the movement of the product along the shaft: the condition of the grain at the moment it enters the shaft affects its condition at the outlet only after a time interval τ has elapsed. The delay may be constant (τ = const) or time-varying:

$$0 \le \tau(t) \le \tau_{max}, \qquad \dot{\tau}(t) \le \mu < 1$$

In practice, the transport delay is directly related to the product feed rate V_f: if the shaft length is L, the time taken for the grain to pass through the shaft can be approximately estimated as

$$\tau(V_f) \approx L / V_f$$

This expression shows the dependence of the delay on the control signal (V_f), and, when linearized about the operating point, yields the relation

$$\Delta\tau \approx -(L / V_{f0}^2) \cdot \Delta V_f$$

that is, increasing the product feed rate reduces the delay but simultaneously shortens the time available for heat and mass transfer — a trade-off that must be taken into account in control synthesis.

The process is also affected by external disturbances:

$$w(t) = [\, T_{env},\ H_{env},\ W_{in} \,]^T$$

where T_{env} — ambient (environmental) temperature; H_{env} — ambient air humidity; W_{in} — moisture content of the incoming product. As a result, the system that accounts for the delay and the disturbances is written in the following form:

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)) + Bu(t) + Ew(t)$$

Linearization of the Nonlinear Model and Derivation of the State-Space Matrices

The system of nonlinear equations from Section 2.3 is reduced to the linear state-space form convenient for formulating the uncertainty and identification problems required in Section 2.7. For this purpose, a nominal (base) operating regime

$x_0 = (T_{g0}, T_{a0}, W_0, H_0)$, $u_0 = (Q_0, G_0, V_{f0})$, $w_0 = (T_{env0}, H_{env0}, W_{in0})$

is chosen, and the state, control, and disturbance variables are introduced as deviations from the nominal values: $\Delta T_g = T_g - T_{g0}$, $\Delta T_a = T_a - T_{a0}$, $\Delta W = W - W_0$, $\Delta H = H - H_0$, and so on. The equations from Section 2.3 are expanded in a Taylor series about the nominal point and truncated to first-order terms (this is the standard linearization procedure, applicable under small-deviation conditions).

For notational purposes, let $k_1 = dk/dT_g|_0$ (the sensitivity of the drying coefficient to temperature) and $k_0 = k(T_{g0})$. This yields the following Jacobian matrices (all coefficients evaluated at the base point):

$$A_0 = \begin{bmatrix} -hA/(m_g c_g) + rk_1/c_g\cdot(W_0 - W_{e0}) & hA/(m_g c_g) & rk_0/c_g & 0 \\ hA/(m_a c_a) & -hA/(m_a c_a) - G_0/m_a & 0 & 0 \\ -k_1(W_0 - W_{e0}) & 0 & -k_0 & 0 \\ (m_g/m_a)k_1(W_0 - W_{e0}) & 0 & (m_g/m_a)k_0 & -G_0/m_a \end{bmatrix}$$

B_0 = [column 1: 0, $1/(m_a c_a)$, 0, 0]; [column 2: 0, $-(T_{a0} - T_{env0})/m_a$, 0, 0]; [column 3: 0, 0, 0, 0]

E_0 = [column 1: 0, G_0/m_a, 0, 0]; [column 2: 0, 0, $k_0\cdot\partial W_e/\partial H_{env}|_0$, $G_0/m_a - (m_g/m_a)k_0\cdot\partial W_e/\partial H_{env}|_0$]; [column 3: 0, 0, 0, 0]

C = [row 1: 0, 0, 1, 0]; [row 2: 0, 1, 0, 0]; D = 0

The delay matrix A_d expresses the dependence of the current state on its value τ time units earlier, arising from the displacement of the grain layer along the shaft; this dependence is manifested primarily in the grain-temperature and moisture channels, since it is precisely these two variables that physically "travel" together with the product:

A_d = diag-type matrix with nonzero entries $a_{d,11}$ (position 1,1) and $a_{d,33}$ (position 3,3); all other entries are zero

Here the coefficients $a_{d,11}$ and $a_{d,33}$ are determined, through the identification procedure described in Section 2.8, from the residence-time distribution of the grain layers in the shaft; the nonzero elements of this matrix are interpreted as a locally linearized, finite-dimensional (lumped) approximation of the spatially distributed heat/moisture dynamics along the shaft (a detailed PDE-level description lies outside the scope of this study and is regarded as a possible direction for future work).

The matrices A_0, B_0, E_0, C, A_d obtained above constitute the direct result of linearizing the four physical balance equations of Section 2.3 and represent the final numerical outcome of the linearization procedure: they are now written as precise linear operators (matrices) with numerical coefficients with respect to the variables x, u, w, and are used directly in the following sections (2.6–2.8) to formulate the uncertainty, identification, and verification problems.

The physical quantities appearing in the matrices are summarized in Table 2.1.

Table 1. Principal parameters of the linearized model

Symbol	Definition	Unit
m_g	Mass of grain in the shaft	kg
c_g	Specific heat capacity of grain	J/(kg·K)
m_a	Mass of air in the shaft	kg
c_a	Specific heat capacity of air	J/(kg·K)
h	Heat transfer coefficient	W/(m²·K)
A	Heat transfer surface area	m²
r	Latent heat of vaporization of moisture	J/kg
k0, k1	Drying coefficient and its temperature sensitivity	1/s, 1/(s·K)
W_e	Equilibrium moisture content	kg/kg
G	Air flow rate (control input)	kg/s
Q	Heat capacity/power (control input)	W
V_f	Product feed (throughput) rate (control input)	m/s
L	Shaft length	m
τ	Transport delay	s
Tenv, Henv, Win	External disturbances	K, kg/kg, kg/kg

The structural block diagram of the model's signal flow (from the input u(t) and disturbance w(t) to the output y(t), via the current state and the state delayed by τ) is presented in Fig. 2.4.

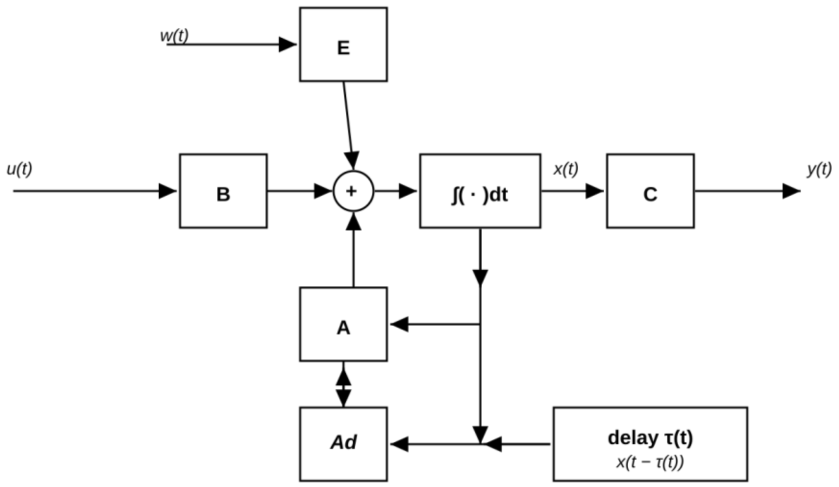

Fig. 4. Structural diagram of the linearized state-space model of the shaft-type dryer.

Thus, the transition from the physical balances presented in Sections 2.3–2.4 to the linear state-space matrices above forms the following complete chain:
energy/mass balance → nonlinear ODE system → linearization → state-space (A_0, B_0, C, E_0, A_d) → identification (Section 2.8) → verification (Section 2.9)

Control Quality Criterion (Objective Function)

The multi-criteria objective function formulated in Chapter 1 (within the scope of the research tasks)

$$J = \alpha_1 J_{moisture} + \alpha_2 J_{energy} + \alpha_3 J_{time}$$

is specified in this chapter through the variables {x(t), u(t), y(t)} of the state-space model obtained above, as follows:

$$J_{moisture} = \int_0^T (W_{out}(t) - W^*_{out})^2\, dt = \int_0^T (y_1(t) - W^*_{out})^2\, dt$$

$$J_{energy} = \int_0^T (\lambda_1 Q^2(t) + \lambda_2 G^2(t))\, dt = \int_0^T u^T(t)\Lambda u(t)\, dt$$

$$J_{time} = T_{drying}(V_f) \approx L / V_f$$

where W^*_{out} — the specified (target) value of the outlet moisture content; $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2)$ — the weighting-coefficient matrix of the energy costs; α_1, α_2, α_3 — the weighting coefficients introduced in Chapter 1 (the methodology for determining them is defined in the

research tasks of Chapter 1). It is important to note that the expression for J_{time} is directly related to the relation $\tau(V_f) \approx L/V_f$ obtained in Section 2.4: increasing the product feed rate reduces J_{time}, but by reducing τ it shortens the time available for heat–mass transfer, which typically increases $J_{moisture}$. The components of J are therefore mutually conflicting, giving rise to a multi-criteria optimization problem.
Thus, the state-space model $\{A(\Delta), A_d, B(\Delta), C(\Delta), E\}$ developed in this chapter serves as the direct mathematical foundation for synthesizing (in Chapter 3) the controller $u = f(x, \Delta, \tau)$ that minimizes the objective function J; the methodology for finding the Pareto-optimal solution of J is the subject of Chapter 3.

References

1. On the Strategy of Actions for the Further Development of the Republic of Uzbekistan. - T.: February 7, 2017, Decree No. PF-4947.
2. Yusupbekov N.R., Mukhiddinov D.P. Fundamentals of modeling and optimization of technological processes. - T.: "Science and Technology", 2018, p. 440.
3. Yusupbekov N.R., Mukhamedov B.I. Control and automation of technological processes. Textbook. – T.: 2017. – 576 p.
4. Yusupbekov A.N., Esonov H.M. Practical importance of automation and control systems sciences -T.Article. International conference "Prospects of geology, mining and metallurgy and oil and gas sectors in the south of the republic".
5. Esonov H.M. Improving the efficiency of executive mechanisms of intellectual control systems. Monograph. -T.: "Termiz publishing center",2024, 160p.
6. Hung, P. V. Maeda, T.; Miyatake, K. Morita, N. Total phenolic compounds and antioxidant capacity of wheat graded flours by polishing method. Food Res. Int. 2009, 42 (1), 185−190.
7. Martínez-Tomé, M. Murcia, M. A. Frega, N.; Ruggieri, S. Jimenez, A. M. Roses, F. Parras, P. Evaluation of antioxidant capacity of cereal brans. J. Agric. Food Chem. 2004, 52 (15), 4690−4699.
8. Chen, Y.-X. Guo, X.-N.; Xing, J.-J. Zhu, K.-X. Effects of tempering with steam on the water distribution of wheat grains and quality properties of wheat flour. Food Chem. 2020, 323, No. 126842
9. Zheng, Z. Wang, S.; Zhang, C. Wu, M.; Cui, D. Fu, X. Gao, L. Li, A. Wei, Q. Liu, Z. Hot air impingement drying enhanced drying characteristics and quality attributes of ophiopogonis radix. Foods 2023, 12 (7), 1441.
10. Piwinska,́ M. Wyrwisz, J. Wierzbicka, A. Effect of micronization of high-fiber oat powder and vacuum-drying on pasta quality. CyTAJ. Food 2016, 14 (3), 433−439.
11. Dong, R. Lu, Z.; Liu, Z. Koide, S. Cao, W. Effect of drying and tempering on rice fissuring analysed by integrating intra-kernel moisture distribution. J. Food Eng. 2010, 97 (2), 161−167.
12. Jha, A. Tripathy, P. P. Influence of solar drying and storage conditions on microstructure, crack propagation and nano-hardness of paddy and wheat. J. Cereal Sci. 2020, 95, No. 103054.

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