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Mathematical Modeling of The Machining Process of White Cast Iron

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Abstract

This paper presents a mathematical model in the form of complex differential equations describing the mechanical cutting of white cast irons. Heat conduction PDE, similarity criteria theory (Peclet, F, D), and mechanical cutting theory were applied. A system of six interrelated equations was developed covering geometry, temperature, forces, plasticity, wear, and heat balance. Conclusion: The model enables justification of tool material requirements for machining white cast irons.

Keywords: White cast iron; mechanical machining; mathematical model; differential equation; temperature; cutting forces; wear; Peclet criterion; WC-Co hard alloy; heat balance.

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1. Introduction

White cast irons are widely used in industry due to their extremely high hardness (HV 4500–7000 MPa) and excellent wear resistance. Typical applications include crusher plates, mining equipment, pumps, and other components. However, these same properties make them one of the most difficult materials to machine. The machining of white cast iron involves complex physicomechanical and thermal phenomena [1–3].

To date, no specialized cutting tool material has been developed specifically for machining white cast iron. Cutting conditions for existing hard alloys of the VK group (VK3M, VK4, VK6, VK8, etc.) remain insufficiently defined [4–6]. This leads to reduced productivity and accelerated tool wear.

To address this problem, a comprehensive mathematical model capable of fully describing the process is required. The aim of this study is to develop and analyze a mathematical model in the form of complex differential equations representing the machining process of white cast iron. The proposed model incorporates the physicomechanical characteristics of the cutting process.

A review of the literature indicates that in-depth analysis of machining processes is essential, particularly with regard to tool wear, heat generation, and surface quality. In a study conducted by Rosa S. D. N. et al., the relationships between tool wear, surface roughness, and cutting power during the turning of compacted graphite iron were investigated experimentally [7]. Their results are important for selecting optimal cutting parameters. Zhu K. reviewed modern approaches to modeling machining processes and emphasized the growing role of digital technologies and intelligent systems [8]. In this context, mathematical modeling is considered a key tool for improving machining efficiency.

In a study by Soori M. et al., methods for predicting cutting tool wear were systematically analyzed. The authors compared the advantages and limitations of analytical, empirical, and artificial intelligence–based models. In particular, the high accuracy of machine learning approaches was emphasized [9]. A monograph by Grzesik W. provides a comprehensive overview of the theoretical foundations, modeling techniques, and practical applications of advanced machining processes for metallic materials [10]. In this work, heat generation, deformation zones, and cutting forces are analyzed on a rigorous scientific basis.

The reviewed literature indicates that comprehensive modeling of machining processes plays a decisive role in extending tool life and improving product quality. At the same time, the integration of mathematical models with experimental results is considered a key direction in modern research. This approach enables the optimization of machining processes and enhances energy efficiency.

2. Methods

The study was conducted based on theoretical approaches. The following principles were applied in the development of the mathematical model:

- The Fourier heat conduction equation was used to calculate the temperature field in the cutting zone;
- The Tyme and Makarov–Mukhin formulations were applied to determine heat generation in the shear plane;
- Similarity theory was employed to evaluate the plasticity parameter BBB using the Pe, F, and D criteria [7];
- Cutting force theory was used to calculate the force components [7–8];
- Wear theory was applied to determine the optimal cutting speed and temperature [9–10].

The workpiece material is eutectic white cast iron

(λ = 80,4 J/(m·s·°C); C_ρ = 7.02×10^6 J/(m^3°C); a = 12×10^{-6} m^2/s).

The cutting tools used were cemented carbide inserts of grades VK6B, VK6, VK6M, and VK6OM. The cutting conditions were as follows

: v=0.05–1.0 m/s; s=0.2×10^{-3} m/rev $s = 0.2 \times 10^{-3}$ m/rev; t=2×10^{-3} m; t=2×10^{-3} m; γ = 12°; α = 12°; φ = 45°.

The developed mathematical model consists of a system of six interrelated equations describing the following key processes: the geometry of the removed layer, temperature distribution in the cutting zone, cutting forces, the plasticity parameter (denoted as BBB), cutting tool wear, and heat balance.

The thickness a_1 and the width b_1 of the removed layer are the primary geometric parameters of the cutting process and are determined by the following expressions:

$$a_1 = s \cdot \sin\varphi\,, \quad b_1 = t / \sin\varphi, \tag{1}$$

$$b = \sqrt{[s \cdot r \cdot (\sin\varphi + \sin\varphi_1)/(\sin\varphi \cdot \sin\varphi_1)]}, \tag{2}$$

Here, s is the feed rate [m/rev]; f_1 and f_2 are the principal and auxiliary angles in the plan; r is the tool nose radius [m]; and t is the depth of cut.

It is well known that the primary physical transformation in the cutting process is governed by temperature variation. Therefore, to derive the mathematical model, a partial differential equation describing heat transfer is employed.

The temperature distribution in the cutting zone is described by the following partial differential equation:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{Q(x,y,t)}{\rho c}, \tag{3}$$

Here, T is the temperature [$^\circ\mathrm{C}$]; aaa is the thermal diffusivity [m^2/s]; Q is the internal heat source [J/s]; ρ is the density [kg/m^3]; and c is the specific heat capacity [J/(kg·°C)].

Boundary conditions are formulated for differential equation (3).

The AB plane corresponds to the shear plane (primary plastic deformation zone):

$$T_A = \tau_r \cdot v \cdot a_1/(\lambda \cdot c_0) \qquad (4)$$

The AC surface corresponds to the rake face (chip–tool friction zone):

$$q_p = \mu_p \cdot N \cdot v/(a_1 \cdot b_1) \quad [J/s]. \qquad (5)$$

The AD surface corresponds to the flank face:

$$q_r = \tau_r \cdot v \cdot (1 - \sin\alpha)/\cos\alpha. \qquad (6)$$

Here, τ_r is the resistance to plastic shear [N/m^2]; v is the cutting speed [m/s]; λ is the thermal conductivity [J/(m·s·°C)]; and μ_p is the coefficient of friction.

The tangential and radial components of the cutting force are expressed as follows:

$$R_i = \tau_r \cdot a_1 \cdot b_1 \cdot \cos(45^0 - \beta_1 + \gamma)/\left[\sin\beta_1 \cdot \cos(\beta_1 - \gamma + \mu_p)\right], \qquad (7)$$

$$R_j = \tau_r \cdot a_1 \cdot b_1 \cdot \sin(45^0 - \beta_1 + \gamma)/\left[\sin\beta_1 \cdot \cos(\beta_1 - \gamma + \mu_p)\right]. \qquad (8)$$

The average coefficient of friction:

$$\mu_p = (tg\beta_1 + tg\gamma)/(1 - tg\beta_1 \cdot tg\gamma), \qquad (9)$$

Here, $\beta_1$is the shear plane angle [rad]; γ is the rake angle of the cutting tool [rad].

The evaluation of the plasticity level is one of the most important aspects of the cutting process; it can be expressed using similarity criteria.

The parameter $tg\beta_1$ (defined as the tangent of the shear plane angle) is given as:

$$tg\beta_1 = m \cdot Pe^n \cdot F^k \cdot D^p \cdot (1 - \sin\gamma)^{\varphi}. \qquad (10)$$

The Péclet and other similarity criteria:

$$Pe = v \cdot a_1/a \text{ (The Péclet criteria)}, \qquad (11)$$

$$F = (\lambda_r/\lambda) \cdot (1 - \sin\gamma)/(\sin\beta \cdot \cos\varepsilon),\ D = a_1/b_1. \qquad (12)$$

The Péclet and other similarity criteria can be described based on Table 1.

Table 1.
Similarity criteria

Criteria	Expression	Meaning
Pe	$v \cdot a_1 / a$	thermal effect of cutting conditions

Criteria	Expression	Meaning
F	$(\lambda_r / \lambda)\cdot(1-\sin\gamma)/(\sin\beta\cdot\cos\varepsilon)$	tool geometry and λ ratio
D	a_1 / b_1	layer cross-section geometry
$tg\beta_1$	$m\cdot Pe^n\cdot F^k\cdot D^p\cdot(1-\sin\gamma)^{\varphi}$	plasticity level

The optimal cutting temperature (from the cobalt liquidus temperature) for determining cutting tool wear is obtained as follows:

$$T_0 = T^1\cdot\sqrt{\lambda_r\cdot C_{qr}/\lambda\cdot C_q}. \qquad (13)$$

The optimal cutting speed is:

$$v_0 = m_1\cdot\left[T_0/f(\lambda, C_q, \tau_r, \delta, \sigma_p)\right]^{1/m}. \qquad (14)$$

Relative linear decay:

$$h^0 = h_r/L_0 = v_0\cdot h_0/L_0, \qquad (15)$$

where: T^1— liquidus temperature of cobalt (1490 °C); δ— relative elongation [%]; σ_p— strength [N/m²].

In thermal equilibrium, mechanical energy is completely converted into thermal energy:

$$R_i\cdot v = Q_c + Q_d + Q_r, \quad [J/s], \qquad (16)$$

Here, $R_i v$ is the total mechanical power; Q_c is the heat carried away by the chip; Q_d is the heat transferred to the workpiece; and Q_r is the heat transferred to the cutting tool.

3. Results and Discussion

Based on the developed mathematical model, calculations were carried out for machining white cast iron using cemented carbides of the VK group.

When machining white cast iron with VK6B grade cemented carbide, the distribution of thermal energy is as follows (Table 2):

The analysis of the table shows that with an increase in cutting speed, the proportion of heat carried away by the chip (Q_c/Q) increases, while the proportion of heat transferred to the cutting tool (Q_r/Q) decreases. This effect is more pronounced for the VK6B cemented carbide with a grain size of 2–5 µm.

Table 2.
Heat balance during machining of white cast iron with VK6B grade cemented carbide

v, m/s	Pe	Q_c, J/s	Q_c/Q, %	Q_d, J/s	Q_d/Q, %	Q_r, J/s	Q_r/Q, %
0,05	1,09	18,23	35,9	24,33	47,9	8,22	16,2
0,12	2,19	52,17	49,6	41,86	39,8	11,15	10,6
0,25	4,25	111,07	57,7	68,15	35,4	13,28	6,9

v, m/s	Pe	Q_c, J/s	Q_c / Q, %	Q_d, J/s	Q_d / Q, %	Q_r, J/s	Q_r / Q, %
0,50	8,70	238,40	65,8	105,42	29,1	18,47	5,1

The calculated results of the optimal cutting speed for VK grade cemented carbides are presented in Table 3.

Table 3.
Optimal cutting speed for VK grade cemented carbides

Cemented carbide	λ_p, J/(m·s·°C)	T_0, °C	v_0, m/s
VK6B	58,5	900	0,0214
VK6	56,7	850	0,0189
VK6M	55,9	800	0,0176
VK6OM	54,7	750	0,0143

The analysis of the results shows that currently recommended VK group cemented carbides do not provide optimal cutting speeds when machining white cast iron. Relatively better performance was demonstrated by the VK6B grade cemented carbide with coarse tungsten carbide grains (v0=0.0214 m/s).

An increase in the thermal conductivity of the cutting tool material (λ_p) leads to a decrease in the shear angle $(\tan\beta_1)$. When the Péclet number Pe>10, the cutting regime changes significantly, resulting in intensified heat flow and increased requirements for the heat resistance of the tool material.

4. Conclusions

Based on the developed model, the following key conclusions can be drawn:

1. An increase in the feed sss and the angles φ and φ_1 leads to an increase in the uncut chip thickness a_1; conversely, an increase in the ratio r/t results in its reduction.
2. An increase in the thermal conductivity of the cutting tool material (λ_r) reduces the shear plane angle while increasing the cross-sectional area of the chip.
3. As the resistance of white cast iron to plastic deformation increases, the temperature on the rake face of the cutting tool rises sharply.
4. When the cutting speed exceeds 0.64 m/s the wear rate of all VK6-type cemented carbides increases significantly.
5. The heat generated in the VK6M tool (with fine grain size, ≤1 µm) is significantly higher than that in VK6B with coarser grains (2–5 µm).
6. The developed mathematical model serves as a theoretical basis for the development of new composite cutting tool materials for machining white cast iron. The use of cemented carbides based on ultra-coarse tungsten carbide grains is recommended.

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