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A Practical Method For Calculating Cylindrical Shells

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ABSTRACT

This article outlines the main provisions of the proposed method of determining the stress-strain state of cylindrical shells. The method can be used to calculate the various sheet structures, such as rectangular and cylindrical steel tanks for the storage of fluid substances.

KEYWORDS

Stress-strain state, strength, stiffness, the tank shell, leaf design, approximation, allowing functions determinant.

INTRODUCTION

Cylindrical shell tanks are widespread structural elements that are part of various metal structures [1]. They have high bearing capacity and rigidity at a relatively low cost of metal. Their widespread adoption is in direct proportion to the availability of effective methods for determining the stress-strain state, assessing strength, stiffness and stability [2-7]. Below is a suggested practical method for solving this problem.

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The stress-strain state of thin shells can be described by means of one or several resolving functions depending on two variables. Finding them is associated with solving a boundary value problem for partial differential equations. The essence of the method under consideration is to reduce this problem to a boundary value problem for a system of ordinary differential equations. This is achieved by appropriate approximation of the required functions of two variables, which allows us to consider it continuous in one variable and discrete in the second.

For approximation, a system of n coordinate lines with a uniform step Δ is applied to the area under consideration (Fig. 1). These lines are considered defining. Their equations are of the form $x = k\Delta$, $k = 1, 2, ..., n$, i.e., along the lines one variable has a certain fixed value. Regions are considered that allow defining lines to be drawn in such a way that they intersect the contour of the region at no more than two points, and the extreme lines touch it.

MATERIALS AND METHODS

Taking one of the variables in the desired function to be alternately equal to the constants corresponding to the equations of the defining lines $x = k\Delta$, we obtain a system of functions of one variable y, which are the values of the sought functions along the defining lines. Let's call them defining functions. They allow you to get an approximate idea of the nature of the change in the desired function and are taken as approximating ones. We assume that the defining functions have a periodic nature of change with a period equal to the distance between the extreme lines tangent to the contour. It follows from what has been said that the desired function of two variables u (x, y) is determined by a system of n functions of one variable $U_1(x_1, y)$, $U_2(x_2, y)$. We combine them into a vector $(x_k, y) = || U_1(x_1, y), U_2(x_2, y) ... U_n(x_n, y) ||$

You can give a slightly different interpretation of the vector, considering it as a function of two variables, having a discrete-continuous character of change, taking this into account when performing various mathematical operations on it. So, when differentiating a vector along the x coordinate, the derivatives should be replaced by finite difference relations, the original partial differential equation should be replaced by a system of ordinary differential equations. They are obtained from the initial one by writing it in turn along the defining lines with the simultaneous replacement of derivatives in the discrete direction by finite-difference relations. The components of the vector are found from the system composed in this way.

t should be emphasized that the number of unknowns and equations exactly coincide and are equal to n. The solution contains arbitrary constants, the number of which depends on the order of the differential equation under consideration. They are found from the boundary conditions of the problem, compiled for the points of intersection of the defining lines with the contour of the region.

Let us note a feature of the solution that arises when the extreme defining lines coincide in some section with the contour of the region. Then the required function is taken as the sum of two functions. Each term is approximated as described above, only the sampling directions are taken to be different, in different variables. This is equivalent to the adoption of two orthogonal systems of defining lines (Fig. 2), where $\Delta_1 = l_1 / (n-1)$; $\Delta_2 = l_2 / (m - 1)$, and the corresponding functions (x_k, y) and (x, y_k). The components of these vectors are found from two independent systems of differential equations. Arbitrary constants included in them are determined from the general system of boundary equations compiled for the points of intersection of both systems of defining lines with the contour of the region.

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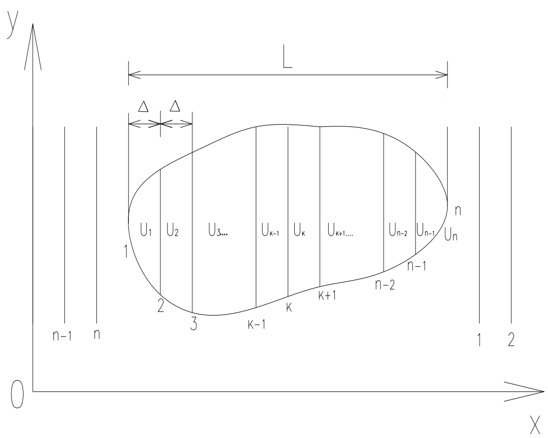

Figure 1. Scheme of drawing the defining straight lines.

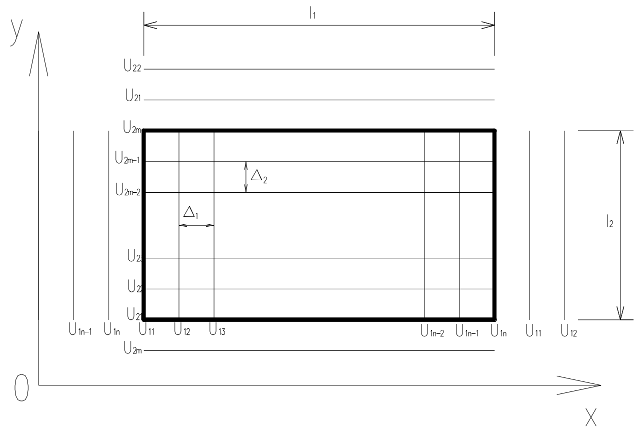

Figure 2. The layout of cross lines

Let us point out an essential feature of the method under consideration, which follows from the assumption of the periodic nature of the change in the determining functions. In this case, the structure of matrices of ordinary differential equations is obtained such that one can find their general solution in analytical form. Let's dwell on this issue in more detail.

First of all, we note that the periodicity of the defining functions makes it possible to obtain the derivatives of the vector in the discrete direction by multiplying it on the left by some matrices: - in left differences; - in the right; - in the central differences. Matrices A and B have the form:

$$A=\begin{Vmatrix} 1 & 0 & 0 & . & . & . & 0 & 0 & -1 \\ -1 & 1 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & -1 & 1 & . & . & . & 0 & 0 & 0 \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ 0 & 0 & 0 & . & . & . & -1 & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & -1 & 1 \\ 0 & 0 & 0 & . & . & . & 0 & 0 & -1 \end{Vmatrix};\ B=\begin{Vmatrix} -1 & 1 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & -1 & 1 & . & . & . & 0 & 0 & 0 \\ 0 & 0 & -1 & . & . & . & 0 & 0 & 0 \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ 0 & 0 & 0 & . & . & . & -1 & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & -1 & 1 \\ 1 & 0 & 0 & . & . & . & 0 & 0 & -1 \end{Vmatrix}:$$

Higher-order derivatives are obtained by sequentially multiplying the corresponding number of times the vector on the left by matrices A or B. Alternating these factors automatically translates even derivatives into central differences with higher accuracy, so it is advisable to use just such an approach to calculating derivatives above the first order. An essential role is played in this case by the properties of the matrix C, which is the product of the matrices A and B, i.e. C = A · B = B · A. By direct multiplication we get:

$$C=\begin{Vmatrix} -2 & 1 & 0 & . & . & . & 0 & 0 & 1 \\ 1 & -2 & 1 & . & . & . & 0 & 0 & 0 \\ 0 & 1 & -2 & . & . & . & 0 & 0 & 0 \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ 0 & 0 & 0 & . & . & . & -2 & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 1 & -2 & 1 \\ 1 & 0 & 0 & . & . & . & 0 & 1 & -2 \end{Vmatrix}:$$

The following relations are obvious:

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$$\underbrace{A\cdot B}_{1}\cdot\underbrace{A\cdot B}_{2}...\underbrace{AB}_{k}=\underbrace{B\cdot A}_{1}\cdot\underbrace{B\cdot A}_{2}...\underbrace{B\cdot A}_{k}=C^{k};$$

$$A\cdot\underbrace{B\cdot A}_{1}\cdot\underbrace{B\cdot A}_{2}...\underbrace{B\cdot A}_{k}=\underbrace{A\cdot B}_{1}\cdot\underbrace{A\cdot B}_{2}...\underbrace{A\cdot B}_{k}\cdot A=A\cdot C^{k}=C^{k}\cdot A;$$

$$B\cdot\underbrace{A\cdot B}_{1}\cdot\underbrace{A\cdot B}_{2}...\underbrace{A\cdot B}_{k}=\underbrace{BA}_{1}\cdot\underbrace{BA}_{2}...\underbrace{B\cdot A}_{k}\cdot B=B\cdot C^{k}=C^{k}\cdot B;$$

Taking these dependencies into account, we write the derivatives of any order

$$\left.\begin{aligned}
&\frac{\partial^{2k}\overline{U}}{\partial x^{2k}}=\frac{1}{\Delta^{2k}}C^{k}\overline{U};\\
&\frac{\partial^{2k+1}\overline{U}}{\partial x^{2k+1}}=\frac{1}{\Delta^{2k+1}}AC^{k}\overline{U}=\frac{1}{\Delta^{2k+1}}C^{k}A\overline{U};\\
&\frac{\partial^{(2k+1)}\overline{U}}{\partial x^{(2k+1)}}=\frac{1}{\Delta^{2k+1}}BC^{k}\overline{U}=\frac{1}{\Delta^{2k+1}}C^{k}B\overline{U}
\end{aligned}\right\}\qquad (1)$$

Let us present the properties of the matrices A, B, C required in what follows. The indicated matrices can be represented in the following form:

$$A=-\frac{1}{2}V\left[\{\lambda_{k}\}+\{\gamma_{k}\}\tilde{E}\right]V;\qquad B=\frac{1}{2}V\left[\{\lambda_{k}\}-\{\gamma_{k}\}\tilde{E}\right]V;\qquad C=V\{\lambda_{k}\}V.\qquad (2)$$

Here V is a square matrix; $\{\lambda_k\}$, $\{\gamma_k\}$ - diagonal matrices. Their elements are equal:

$$V_{ik}=\frac{\sqrt{n}}{n}\left(\cos\frac{2ik\pi}{n}+\sin\frac{2ik\pi}{n}\right);\qquad \lambda_{k}=-4\sin^{2}\frac{k\pi}{n};\qquad \lambda_{k}=-4\sin^{2}\frac{k\pi}{n}\cos\frac{k\pi}{n}.\qquad (3)$$

Matrix $\tilde{E}$ has the form:

$$\overset{A}{E}=\left\|\begin{array}{cccccccccccc}
 & & & & & & & & & \cdot & & \\
 & & & & & & & & 1 & \cdot & & \\
 & & & & & & & 1 & & \cdot & & \\
 & & & & & & \cdot & & & \cdot & & \\
 & & & & & \cdot & & & & \cdot & & \\
 & & & & \cdot & & & & & \cdot & & \\
 & & & \cdot & & & & & & \cdot & & \\
 & & 1 & & & & & & & \cdot & & \\
 & 1 & & & & & & & & \cdot & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & & & & & & & & & \cdot & 1 & \\
 & & & & & & & & & \cdot & &
\end{array}\right\|\qquad (4)$$

The matrix V has the property

$V\cdot V=E$, т. е. $V=V^{-1}$, (5)

on which the finding of inverse matrices for products of the form *VDV*:

$[VDV]^{-1} = VD^{-1}V.$ (6)

Matrix D must be nonsingular.

Let us dwell on the use of the described method for solving two-dimensional linear partial differential equations. When using a discrete-continuous approximation of the desired function U (x, y), any equation of this type, taking into account (1), can be reduced to the form

$$\left[\sum_{j=o}^{j=m} K_{lj}C^j + \sum_{j=o}^{j=r} K_{2j}C^j A + \sum_{j=o}^{j=p} K_{3j}C^j B\right] U = \bar{q}. \qquad (7)$$

Based on dependencies (2), (5), this expression is reduced to the form

$$\mathrm{VDV} \cdot \bar{U} = \bar{q}. \qquad (8)$$

Taking into account dependences (2), (4), it can be argued that the matrix D in its structure is a cross diagonal matrix:

$$\theta = \left\| \begin{array}{cccccccccc} d_1 & & & & & d_1 & \cdot & & & \\ & d_2 & & & d_2 & & \cdot & & & \\ & & \ddots & & \cdot & & \cdot & & & \\ & & & \cdot & & & \cdot & & & \\ & & \cdot & & \ddots & & \cdot & & & \\ d_{n-1} & & & & & d_{n-1} & \cdot & & & \\ \cdot & \cdots & \cdots & \cdots & \cdots & \cdots & \cdot & \cdots & \cdots & \cdots \\ & & & & & & \cdot & d_n & + & d_n \\ & & & & & & \cdot & & & \end{array} \right\|$$

For such matrices, it is easy to find the inverse matrix, and therefore, a solution to system (8) can be obtained $\bar{U} = V \cdot D^{-1} V \cdot \bar{q}.$

Let's consider using the method for calculating thin cylindrical shells. When using three displacements of the middle surface of the shell as resolving functions, the system of equations has the form

$$\left(\frac{\partial^2}{\partial x^2} + -\frac{k_1}{R}\frac{\partial^2}{\partial \theta^2}\right) U_x + \frac{k^2}{R}\frac{\partial^2 U_\theta}{\partial x \partial \theta} + \frac{\nu}{R}\frac{\partial U_z}{\partial x} = -\frac{1-\nu^2}{Eh} q_x;$$

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$$\frac{k_2}{R}\frac{\partial^2 U_x}{\partial x \partial\theta}+\left(k_1\frac{\partial^2}{\partial x^2}+-\frac{1}{R^2}\frac{\partial^2}{\partial\theta^2}\right)U_\theta+\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(1-\frac{h^2}{12}\nabla^2\right)U_z=-\frac{1-\nu^2}{Eh}q_\theta;$$

$$\frac{\nu}{R}\frac{\partial U_x}{\partial x}+\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(1-\frac{h^2}{12}\nabla^2\right)U_\theta+\left(\frac{1}{R^2}+\frac{h^2}{12}\nabla^4\right)\overline{U}_z=-\frac{1-\nu^2}{Eh}q_z;$$

$$k_1=\frac{1}{2}(1-\nu); \qquad k_2=\frac{1}{2}(1+\nu). \qquad (9)$$

Taking the sought-for functions continuous in x, directing the x-axis parallel to the generator, and discrete in, and also expressing the derivatives with respect to x in terms of the differentiation operator S, we find

$$VD_1V\overline{U}_x+k_2ASR\overline{U}_\theta+\nu RSE\overline{U}_z=\overline{P}_x;$$

$$k_2RSB\overline{U}_x+VD_2V\overline{U}_\theta+BVD_3V\overline{U}_z=\overline{P}_\theta; \qquad (10)$$

$$\nu RSE\overline{U}_x+AVD_3\overline{U}_\theta+VD_4\overline{U}_z=\overline{P}_z;$$

Here $\overline{P}_x$, $\overline{P}_\theta$, $\overline{P}_z$ are expressions obtained in the right-hand sides of the system of equations when transforming the original equations. They depend on the loads and the initial values of the sought functions.

Elementsofdiagonalmatrices

$$d_{1k}=(SR)^2-k_1\beta_k^2; \quad d_{2k}=k_1(SR)^2-\beta_k^2; \quad d_{3k}=-\alpha^2(SR)^2+\left(1+\alpha^2\beta_k^2\right);$$

$$d_{4k}=-\alpha^2(SR)^4-2\alpha^2(SR)^2-\left(1+\alpha^2\beta_k^4\right); \qquad \alpha=h/12R^2,$$

The solution to the system (10) can be written as follows:

$$\overline{U}_x=\Delta_0^{-1}\Delta_x; \quad \overline{U}_\theta=\Delta_0^{-1}\Delta_\theta; \quad \overline{U}_z=\Delta_0^{-1}\Delta_z. \qquad (11)$$

The determinants included here have the form

$$\Delta_0=\begin{vmatrix} VD_1V & k_2ASR & \nu RSE \\ k_2RSB & VD_2V & BVD_3V \\ \nu RSE & AVD_3V & VD_4V \end{vmatrix}.$$

Determinants Δ_x, Δ_θ, Δ_z are obtained from Δ_0 replacing the first, second and third columns in it with columns || $\overline{P}_x$, $\overline{P}_\theta$, $\overline{P}_z$ ||. Expanding the determinant Δ_0, we get $\Delta_0=k_1\alpha^2R^2VD_0V$.

The elements of the diagonal matrix are the eighth order polynomial of the operator S:

$$d_{0k} = S^8 \frac{4\beta_k^2}{R^2} S^6 + \frac{1}{R^4}\left[\frac{1-\nu^2}{\alpha^2} - 2(2+\nu)\beta_k^2 + 6\beta_k^4\right] S^4 + \frac{1}{R^6}\left[2(3+\nu)\beta_k^4 - 4\beta_k^6\right] S^2 - \frac{1}{R^8}\left(2-\beta_k^2\right)\beta_k^6.$$

The inverse matrix Δ_0^{-1} we find from the expression:

$$\Delta_0^{-1} = -\frac{1}{k_1\alpha^2 R^2} V D_0^{-1} V = \frac{1}{k_1\alpha^2 R^2} V \left\{\frac{1}{d_{0k}}\right\} V.$$

Determinants Δ_x, Δ_θ, Δ_z are also polynomials of the operator S. Their order is less than eighth. Therefore, their works on Δ_0^{-1} containing regular fractions of the operator S. They correspond to functional expressions, which are linear combinations of eight hyperbolic-trigonometric functions

$$\frac{1}{b_k} sha_k x \sin b_k x; \quad \frac{1}{b_k} cha_k x \sin b_k x; \quad sha_k x \cos b_k x; \quad cha_k x \cos b_k x;$$

$$\frac{1}{\tilde{b}_k} sh\tilde{a}_k x \sin \tilde{b}_k x; \quad \frac{1}{\tilde{b}_k} sh\tilde{a}_k x \sin \tilde{b}_k x; \quad sh\tilde{a}_k x \cos \tilde{b}_k x; \quad ch\tilde{a}_k x \cos \tilde{b}_k x;$$

Here a_k, $\tilde{a}_k$, b_k, $\tilde{b}_k$ — real and imaginary parts of the roots of the polynomial d_{ok}.

Taking into account the above, in formulas (11) we find expanded expressions for $\overline{U}_x$, $\overline{U}_\theta$, $\overline{U}_z$:

$$\overline{U}_x = \sum_{j=0}^{j=1} V\{f_{kj}\} V \overline{U}_{x_0}^{j} + \sum_{j=0}^{j=1} AV \cdot \{f_{kj}\} V U_{\theta_0}^{j} + \sum_{j=0}^{j=3} V\{\psi_{kj}\} V \overline{U}_{z_0}^{j} - V\int_0^x \{f_{k2}(x-t)\}\overline{Q}_x(t)dt -$$

$$- AV\int_0^x \{\varphi_{k2}(x-t)\}\overline{Q}_x(t)dt + V\int_0^x \{\psi_{k2}(x-t)\}\overline{Q}_z(t)dt.$$

The expressions for $\overline{U}_\theta$ и $\overline{U}_z$. By the known ratios, after determining the displacements, one can find the efforts. In displacement expressions, arbitrary constants — $\overline{U}_{x_0}, \overline{U}_{x_0}, \overline{U}_{\theta_0}, \overline{U}_{\theta_0}^{`}, \overline{U}_{z_0}, \overline{U}_{z_0}, \overline{U}_{z_0}^{``}, \overline{U}_{z_0}^{```}$, their total number is 8p. They are determined from the boundary conditions, which are compiled for the points of intersection of the defining straight lines with the contour of the median surface. The direction of the defining straight lines is taken to coincide with the generatrices of the middle surface. Each of them intersects the contour of the latter at two points. Since at each point of the shell contour four boundary conditions must be satisfied, the total number of boundary equations will be equal to 8n, i.e., it coincides with the number of equations (Fig. 3).

The solutions (11) obtained for individual shells make it possible to construct an algorithm for calculating structures that are a combination of these structural elements. In this case, for each of them, a general solution is written based on expressions (11), after which a system of algebraic equations is drawn up, from which arbitrary constants included in all these solutions are found. They are written based on the conditions for pairing individual elements with each other and boundary conditions.

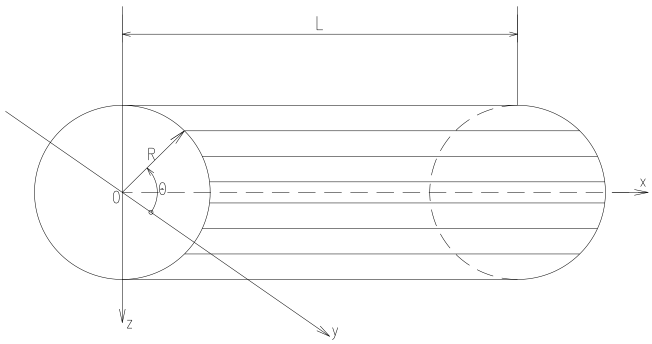

Figure 3. The layout of the defining straight lines for a closed cylindrical shell

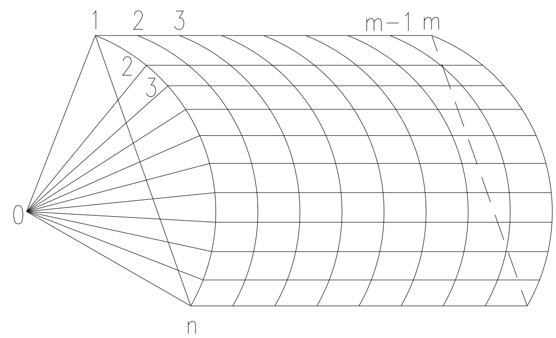

Figure 4. Scheme of drawing cross lines for a cylindrical panel

The general solution for an open cylindrical shell can be obtained by using a system of cross lines, which are applied in the longitudinal and circular directions (Fig. 4). The required function is taken to be the sum of the two functions. Each of them has a system of defining lines.[6,7] Arbitrary constants are found from the conditions on the contour, which are formed at each point of intersection of both systems of lines with the shell boundary. In the presence of a diaphragm (see Fig. 4), it is necessary for it, as for a plate, to write down the general solution and, along the line of contact with the shell, draw up the conjugation conditions, which will be included in the general system of equations for determining arbitrary constants.

CONCLUSION

The proposed method for determining the stress-strain state of individual cylindrical shells can be successfully used in the calculation of various sheet metal structures, such as rectangular and cylindrical tanks, bunkers, box structures.

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