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Statistical Analysis and Forecasting of The Dynamics of Pollutant Emissions into The Atmosphere in The Republic of Uzbekistan

Fayziyev Axtam Asraevich

Candidate of Physical and Mathematical Sciences, Acting Professor, Tashkent University of Economics and Pedagogy, Republic of Uzbekistan

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Abstract: Air pollution is one of the most serious environmental threats to human health. In this article, using the statistical analysis method of time series, the statistical regularity of the dynamic series yt¯\bar{y_t}yt¯ — the average amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan — is studied (based on data from the State Statistics Committee of the Republic of Uzbekistan for the period 2011–2022). With a 95% confidence level, point and interval estimates of the average amount of atmospheric pollutant emissions in Uzbekistan are constructed, obvious trends are identified, and forecasts are made for the following years. Using the Durbin-Watson statistical criterion, it is established that the average amount of pollutants emitted into the atmosphere has autocorrelation dependence.

The applied methods of processing and analyzing dynamic series, after testing, can be used in the research of graduate students and scientific researchers.

Keywords: Discrete, pollutants, substances, atmosphere, dynamic, seasonality, component, linear, least squares, normal, hypothesis, autocorrelation, skewness, kurtosis.

Introduction: In almost every field, there are phenomena that are important to study in their development and change over time. For example, one may seek to predict the future based on past data, control a process, or describe the characteristic features of a series based on a limited amount of information. In time series processing, the methods largely rely on those developed by mathematical statistics for dynamic distribution series. Currently, statistics has a wide variety of time series analysis methods ranging from the most elementary to quite complex ones ([1, 2, 3, 4]).

METHODS

This study involves the processing and analysis of the amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan during the observation period from 2011 to 2022, treated as a discrete time series. The study of the dynamics of atmospheric pollutant emissions globally, and in particular in Uzbekistan, plays an important role in environmental science.

In the general case, a time series $\overline{y}_t$— consists of four components:

1. Trend
2. Fluctuations around the trend
3. Seasonal effect
4. Random component

This study uses time series processing and analysis methods such as: trend determination methods, normality and randomness tests, autocorrelation check, moving average method, finite differences method, least squares method, Durbin-Watson criterion, and others.

Using time series statistical analysis methods, the quantity of atmospheric pollutant emissions in the Republic of Uzbekistan is estimated and forecasted.

The study and analysis of dynamic series has been addressed in the works of Anderson [1], Kendall [2], Tikhomirov [3], Sulaimanov [4], Fayziyev [5], and others.

RESULTS AND DISCUSSION

Let us assume that the quantity of pollutants emitted into the atmosphere in the Republic of Uzbekistan over the observation period 2011–2022 forms a discrete time series. Using the above-mentioned time series statistical analysis methods, we construct point and interval estimates for the amount of atmospheric pollutants, determine the evident trend type of this process, and forecast it for subsequent years. We also test various statistical hypotheses related to this process.

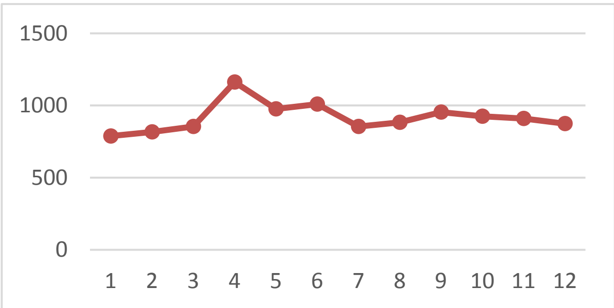
In Figure 1, based on empirical data (Table 1, Column 3), the dynamics of atmospheric pollutant emissions in the Republic of Uzbekistan are geometrically represented as follows:

a) point plot,

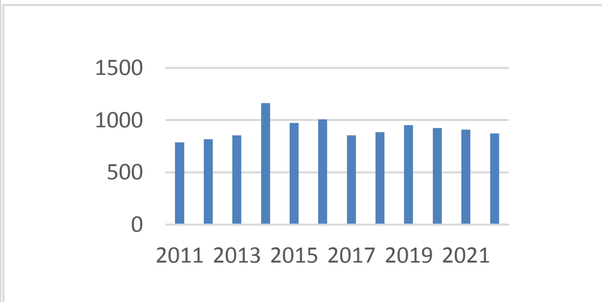
b) histogram,

c) pie chart,

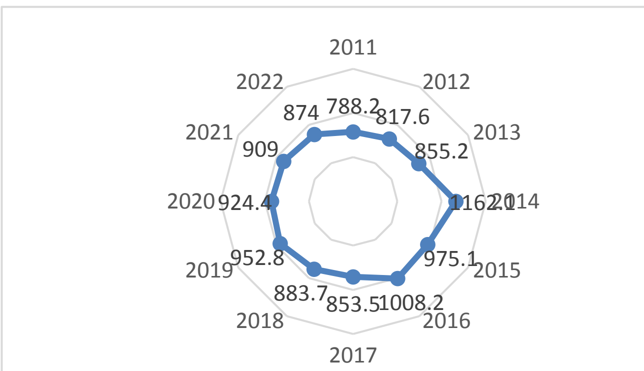
d) radar (spider) chart.



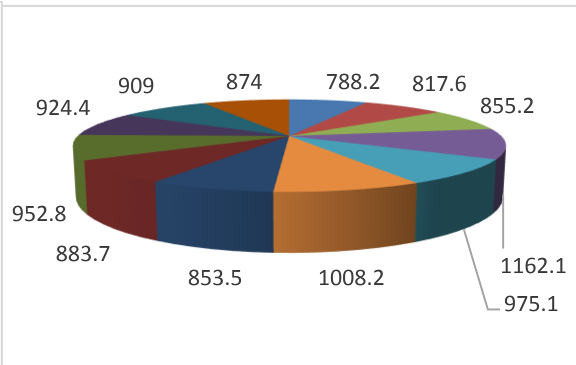
a)



b)



c)



d)

Figure 1.

The geometric representation of the empirical data in a coordinate system provides grounds, at the first approximation, to hypothesize that the trend component of this process (the general direction of its development) has a linear dependence of the form:

yt=a+bty_t = a + btyt=a+bt

where the unknown parameters aaa and bbb are determined by the **method of least squares**, i.e., based on empirical data by solving the following system of normal equations:

$$\begin{cases} a_0 T + a_1 \sum t = \sum y_t \\ a_0 \sum t + a_1 \sum t^2 = \sum y_t t \end{cases} \qquad (1)$$

By solving the system of equations (1) and using the calculations based on Table 1, we obtain:

$\sum y_t$ 11003,8 thousand tons, $a_0 = \frac{1}{T} \sum y_t = \frac{11003,8}{12} = 916,98$ thousand tons

$a_1 = \frac{1}{\sum t^2} \sum y_t t = \frac{5889,6}{146} = 40,34$ .From this, we determine the **linear trend equation** (tendency) for the amount of pollutants emitted:

This equation serves as the **estimated trend of the time series** for the pollutant emissions in the Republic of Uzbekistan.

These calculations form the basis for determining the time series trend.

Table -1

1	2	3	4	5	6	7
N	Year of Observation	y_t thousand tons	t	t^2	$y_t\, t$	$y_t\, t^2$
1	2011	788,2	-5	25	-3941	19705
2	2012	817,6	-4	16	-3270,4	13081,6
3	2013	855,2	-3	9	-2565,6	7696,8
4	2014	1162,1	-2	4	-2324,2	4648,4
5	2015	975,1	-1	1	-975,1	975,1
6	2016	1008,2	0	0	0	0
7	2017	853,5	1	1	853,5	853,5
8	2018	883,7	2	4	1767,4	3534,8
9	2019	952,8	3	9	2858,4	8575,2
10	2020	924,4	4	16	3697,6	14790,4
11	2021	909	5	25	4545	22725
12	2022	874	6	36	5244	31464
Total		11003,8	6	146	5889,6	128050

Pollutants Emitted into the Atmosphere of the Republic of Uzbekistan [1, 2, 3, 4, 5]::

$$y(t) = 40,34\, t + 916,98 \qquad (2$$

In particular, by substituting t=3t = 3t=3 into Equation (2), we obtain the **expected amount of pollutants emitted into the atmosphere** of the Republic of Uzbekistan in the year **2025**, which is on average **1,038 thousand tons**.

Using statistical criteria ([1]–[5]), it was established that in Equation (2), the **null hypothesis** H0:a1=0H_0: a_1 = 0

$y(t) = a_1 t + a_0$ is rejected and the **alternative hypothesis** H_0 : $a_1 = 0$ отвергается и принимается альтерна-тивная гипотеза H_1 : $a_1 \neq 0$ с is accepted at a **significance level of** α=0.05\alpha = 0.05α=0.05. Therefore, the amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan exhibits a **linear trend**.

Autocorrelation represents the correlation dependence between subsequent and preceding members of a time series. We test for the presence of **autocorrelation** in the average amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan, that is:

Yt=ρYt−1+et,Y_t = \rho Y_{t-1} + e_t,

$$Y_t=\rho Y_{t-1}+\varepsilon_t,$$

where ρ = Cov(Y_t,Y_{t+1}) = M$[(Y_t - \bar{y}_t)(Y_{t+1} - \bar{y}_t)]$.

For further analysis, it is necessary to calculate the following **finite differences** based on the observed data. We denote:, $\Delta Y_t = Y_{t+1} - Y_t$, $\Delta^2 Y_t = \Delta Y_{t+1} - \Delta Y_t$, $\Delta^3 Y_t = \Delta^2 Y_{t+1} - \Delta^2 Y_t$.

According to Table 2, the following are calculated:

$$V_k = \frac{\sum_{t=k}^{T} \left(\Delta^k y_t\right)^2}{(T-k)C_{2k}^{k}} \qquad (3)$$

The **coefficients of variation of the differences** are calculated, and it is established that

V1≈V2≈V3.V_1 \approx V_2 \approx V_3.V1≈V2≈V3.

Consequently, the **first-order finite differences eliminate the linear trend**.

Let us now construct a table for the calculation of finite differences.

Table 2 – Calculation of Finite Differences

	2	3	4	5	6	7	8	9
Year of Observation	Y(t) thousand tons	Y_t^2	ΔY_t	ΔY_t^2	$\Delta^2 Y_t$	$\Delta^2 Y_t^2$	$\Delta^3 Y_t$	$\Delta^3 Y_t^2$
2011	788,2	621259,2						
2012	817,6	668469,8	29,4	864,36				
2013	855,2	731367	37,6	1413,76	8,2	67,24		
2014	1162,1	1350476	306,9	94187,61	269,3	72522,49	261,1	68173,21
2015	975,1	950820	-187	34969	-493,9	243937,2	-763,2	582474,2
2016	1008,2	1016467	33,1	1095,61	220,1	48444,01	714	509796
2017	853,5	728462,3	-154,7	23932,09	-187,8	35268,84	-407,9	166382,4
2018	883,7	780925,7	30,2	912,04	184,9	34188,01	372,7	138905,3
2019	952,8	907827,8	69,1	4774,81	38,9	1513,21	-146	21316
2020	924,4	854515,4	-28,4	806,56	-97,5	9506,25	-	18604,96

							136,4	
2021	909	826281	-15,4	237,16	13	169	110,5	12210,25
2022	874	763876	-35	1225	-19,6	384,16	-32,6	1062,76
Total	11003,8	10200748	85,8	164418	-64,4	446000,4	-27,8	1518925

Table 3 – Data for Calculating Autocorrelation Indicators

1	2	3	4	4	5	6	7
N п/п	Year of Observation	y_t – thousand tons	$Y_t \cdot Y_{t+1}$	$Y_t \cdot Y_{t+2}$	$Y_t \cdot Y_{t+3}$	$Y_t \cdot Y_{t+4}$	$Y_t \cdot Y_{t+5}$
1	2011	788,2					
2	2012	817,6	644432,3				
3	2013	855,2	699211,5	674068,6			
4	2014	1162,1	993827,9	950133	915967,2		
5	2015	975,1	1133164	833905,5	797241,8	768573,8	
6	2016	1008,2	983095,8	1171629	862212,6	824304,3	794663,2
7	2017	853,5	860498,7	832247,9	991852,4	729913,2	697821,6
8	2018	883,7	754238	890946,3	861695,9	1026948	755740,2
9	2019	952,8	841989,4	813214,8	960613	929075,3	1107249
10	2020	924,4	880768,3	816892,3	788975,4	931980,1	901382,4
11	2021	909	840279,6	866095,2	803283,3	775831,5	916453,8
12	2022	874	794466	807925,6	832747,2	772353,8	745959
totla		11003,8	9425971	8657058	7814589	6758980	5919269

Using Table 3 and formulas from the literature [1, 2, 3, 4, 5], the **autocorrelation coefficients RLR_LRL** are determined for **lags L=1,2,3,4,5L = 1, 2, 3, 4, 5L=1,2,3,4,5** (where **LLL** is the **lag**, i.e., the time shift — the time interval by which one phenomenon lags behind another related to it):

$$R_L = \frac{\sum_{t=1}^{N-L} Y_t Y_{t+L} - \dfrac{\sum_{t=1}^{N-L} Y_t \sum_{t=L+1}^{N} Y_t}{N-L}}{\sqrt{\left[\sum_{t=1}^{N-L} Y_t^2 - \dfrac{\left(\sum_{t=1}^{N-L} Y_t\right)^2}{N-L}\right]\left[\sum_{t=L+1}^{N} Y_t^2 - \dfrac{\left(\sum_{t=L+1}^{N} Y_t\right)^2}{N-L}\right]}} \quad (4)$$

A **significant deviation of the autocorrelation coefficient RLR_LRL** from zero provides grounds to assume that there is a **substantial autocorrelation dependence** in the average amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan.

On the other hand, let us test the **hypothesis of the existence of autocorrelation** in the average amount of atmospheric pollutant emissions in the Republic of Uzbekistan using the **Durbin–Watson test**:

$$d = \sum_{t=1}^{T-1} (Y_{t+1} - Y_t)^2 / \sum_{t=1}^{T} Y_t^2 . \qquad (5)$$

Using formula (5), we calculate: (5) $d_{наб} = 0,006$ and compare it with the **critical value** с $d_{крит} = 1,22$ from the table values [1], [2], [5]. Since $d_{наб}$ =0,006 < $d_{крит} = 1,22$. it follows that, with **95% confidence**, the **Durbin–Watson test** confirms that the **average amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan** exhibits an **autocorrelated dependence**: $Y_t=\rho Y_{t-1}+\varepsilon_t$. Therefore, the average amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan **this year depends on the emissions from previous and subsequent years**.

Testing the Statistical Hypothesis of Normality For the variable ,

$\bar{y}_t$ the average amount of pollutants emitted into the atmosphe

$$H_0 : P(\bar{y}_t < x) = \Phi_{a,\sigma}(x), \qquad H_1: P(\bar{y}_t < x) \neq \Phi_{a,\sigma}(x)$$

The **significance level** is chosen as α=0.05\alpha = 0.05α=0.05 (see Table 4).

Then, using the following formula, we construct the **confidence interval estimate** for the **average amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan**:

$$\bar{Y}_{T+i} - t(T-2;\alpha)\bar{\sigma}_y \le a_0 + a_1(T+i) \le \bar{Y}_{T+i} + t(T-2;\alpha)\bar{\sigma}_y \quad (6)$$

$$\text{were} \quad \bar{\sigma}_y = \bar{\sigma}\left[\frac{\frac{1}{T} + \left(\frac{T-1}{2} + i\right)^2}{\sum_{i=1}^{T}(t-\bar{t})^2}\right]^{0.5};$$

The **critical value** $t_{крит.} = t(T-2;\ \alpha)$ is determined from the **Student's t-distribution table** (see [5], p. 181).

Using **formula (6)**, we determine the **confidence interval estimate** for $\bar{y}_t$— the **average annual amount of pollutants emitted into the atmosphere** in the Republic of Uzbekistan, **with a probability of 0.95**:

:

(853,44; 980,52) thousand tons

Based on sample data and using the **software package X7.2019** and **Microsoft Excel** ([4], [5], [6]), we compute the **statistical characteristics** of $\bar{y}_t$— the **average annual amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan** (see **Table 4**).

Estimation of the Main Parameters of the Time Series

Table 4

Выборочные характеристики	Оценки выборочные характеристик
Average annual amount of pollutants emitted $\bar{y}_t$ – thousand tons	916,98 ≈ 917
Variance	10040,60
Standard deviation σ_T	100,20
Coefficient of variation v (%)	10,93 %
Skewness A_{ς}	1,30
Kurtosis $E_{K\varsigma}$	2,38
Standard error of the mean $\bar{y}_T$, m_y	$m_y=\frac{\sigma_y}{\sqrt{n}} = 28,88$
Maximum error of the mean m'_y	m'_y= t m_y = 2,20 · 28,88 = 63,54
Error of the standard deviation σ_T	$m_\sigma=\frac{\sigma}{\sqrt{2n}}=\frac{100,20}{4,90} = 20,45$
Confidence interval estimate (95%) $\bar{y}_T \pm$	$\bar{y}_T \pm$ t m_y = 916,98 ± 63,54

tm_y	**(853,44; 980,52) тысяча тонна**
Test of statistical hypothesis $H_0 : P(\bar{y}_t < x) = \Phi_{a,\sigma}(x)$	95% The null hypothesis H_0 is accepted with a given level of confidence.

CONCLUSIONS

Based on the above statistical analyses, the dynamics of $\bar{y}_t$ — the **average annual amount of pollutants emitted into the atmosphere** in the Republic of Uzbekistan — considered as a **discrete time series**, with **95% confidence** (Table 4), the following conclusions can be drawn:

1. **Point and interval statistical estimates** have been constructed for $\bar{y}_t$ — — the average annual amount of pollutants emitted into the atmosphere in the Republic of Uzbekistan. In particular, the forecasted average annual amount of atmospheric pollutants for the year **2023** is expected, with **95% confidence**, to fall within the interval: **(853,44; 980,52) тысяча тонна** ;

2. The **explicit type of trend** has been determined and its **linearity established**, with the following equation:

$$\boldsymbol{y(t) = 40,34\,t + 916,98};$$

3. Based on the **Durbin–Watson criterion**, it was established that the **average annual amount of pollutants emitted into the atmosphere** in the Republic of Uzbekistan exhibits an **autocorrelated dependence**:$Y_t=\rho Y_{t-1}+\varepsilon_t$, где $\rho = Cov(Y_t,Y_{t+1}) = M[(Y_t - \bar{y}_t)(Y_{t+1} - \bar{y}_t)]$ i.e., the volume of pollutants emitted this year depends on the amounts from previous and following years.

4. It has been shown that the **dynamics of the average amount of pollutants** in Uzbekistan **form a non-stationary time series**.;

5. The **high level of atmospheric pollution** globally, and in particular in the Republic of Uzbekistan, warns humanity of the urgent need to **take immediate action** to reduce pollution levels. This requires **coordinated efforts** from **local, national, and regional authorities** in sectors such as **energy, transportation, waste management, urban planning, and agriculture**.

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