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Perfect numbers and their formula. Euclid and Euler's approach. Do odd perfect number exist?

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Abstract: Perfect numbers, one of the fundamental concepts of mathematics have been focus of mathematicians attention science ancient times. This article discusses the concept of perfect numbers, their identification formulas, the Euclid and Euler approaches, as well as of the most debated problems the existence of odd perfect numbers.

Keywords: Perfect numbers, fundamental concepts

Introduction: Perfect numbers are a mysterious aspect of mathematical beauty. Throughout the history of mathematics, perfect numbers have held a special place. Euclid, in his famous work Elements, outlined the principles of the formation of perfect numbers, while Euler expanded this theory further and established a solid scientific foundation for the role of perfect numbers in modern mathematics. Additionally, Mersenne prime numbers play a significant role in the formation of perfect numbers. To this day, all identified perfect numbers are even, and the question of the existence of odd perfect numbers remains open. Perfect numbers are not only important for theoretical mathematics but also hold significance for the overall development of number theory. Therefore, studying this topic in greater depth is beneficial not only from a historical perspective but also from the standpoint of modern

mathematical analysis.

Perfect numbers are defined as numbers that are equal to the sum of all their positive divisors excluding themselves.

Definition: For a positive number N to be a perfect number, the following equation must be satisfied:

σ(N) − N = N , there is σ(N)-the sum of all divisors of N.

In addition, the sum of the divisors of a perfectnumber doubles: (N)=2*N

Examples:

1. 6 : Divisors 1, 2, 3. Sum 1+2+3=6

2. 28 : Divisors 1, 2 , 4 , 7 , 14. Sum 1+2+4+7+14=28

Around 300 BC, Euclid demonstrated that if 2p-1 is prime, then the number $(2^{p-1}) * (2^{p}-1)$ perfect. The first four perfect numbers were the only numbers known to ancient Greek mathematics2.

In mathematics, Nicomachus found 8128 a thousand years ago3. At the same time, there are 20 even perfect numbers. Another condition for a number to be even is that it must be divisible by a prime number p and 2p-1. Here exactly p and 2p-1 must be prime4.

In the 18th century, Leonhard Euler delved deeper into Euclid's famous formula about prime numbers and, based on this formula, proved the following result:

Given N, it consists of two parts:

(2p-1) and (2p-1) . If (2p-1) is a prime number, the divisors of N are as follows:

1.The divisors of (2p-1) :

These are 1, 2, 22, … , 2p-1 , with a total of p divisors.

2.The divisors of (2p-1) : since 2p-1 is a prime number, its divisors are only 1 and (2p-1).

3.All divisors of N :

{ 1, 2, 22, ..., (2p-1), (2p - 1), 2*(2p- 1), (22)*(2p - 1), ..., (2p-1)*(2p - 1)}

The number of divisor is 2*p.

Division Summation Numbers.

Finding the sum of the divisors of numbers.

1-group: {1, 2, 22,…, 2p-1}

This group's sum is calculated using the formula of the geometric series:

S1=1+2+22+…+(2p-1)=2p-1

2-group: {(2p-1), 2*(2p-1), (22)*(2p-1), … ,(2p-1)*(2p-1)}

We write this group in general form:

S2=(2p-1)*(1+2+22+…+2p-1)

We have S1=2p-1

S2=(2p-1)*(2p-1)=(2p-1)2

Calculate the sum of all divisor

The sum of all divisors is equal to S1 and S2: σ(N)=S1+S2=(2p-1)*(2p-1)2

We ara compare σ(N) with 2*N:

In formula N=(2p-1)*(2p-1)

Then: 2*N=2*(2p-1)*(2p-1)=(2p)*(2p-1)

Let's check it:

σ(N)=2p-1+(2p-1)*(2p-1)

This is also equal to 2*N, that is σ(N)=2*N

Even perfect numbers correspond exclusively to Euclid's formula, that is1:

N=(2p-1)*(2p-1)

Euler made a significant contribution to the theory of perfect numbers. He mainly studied the properties of perfect numbers and the mathematical operations related to them in greater depth. In Euler's work Introductio in Analysin Infinitorum, a lot of information about perfect numbers is presented. Among Euler's important ideas are studies on whether perfect numbers are even or odd. Euler, in particular, provided a detailed analysis of even perfect numbers. Another important idea presented by Euler is that if 2p-1 is a prime number, then (2p-1)*(2p-1) is a perfect number. This forms the foundation of Euler's research on perfect numbers2.

Moreover, Euler studied the existence of odd perfect numbers. It is known that any natural number can be expressed as a product of its prime factors.

N= p1a * p2b * ... * pnz

The strength of this function lies in the fact that it can be expressed as a product of its components. It is a multiplicative function3.

σ(N)=σ(p1a * p2b * ... * pnz)=σ(p1a) * σ(p2b) * … * σ(pnz)

Such as,

σ(20)=σ(22)*σ(5)=(1+2+4)*(1+5)=42

Euler proved that every even number adheres to the Euclidean algorithm.

This Euclid-Euler theorem solved a 1600 years old problem. If N is such an odd perfect number that exists, it is (N)=2*N . We can write N other shape N=

p1a * p2b * ... * pnz

σ(N)=σ(p1a * p2b * ... * pnz)=σ(p1a) * σ(p2b) * … * σ(pnz)=2*N

The point to note is that if a prime number has an odd exponent, its sigma will be even:

σ(7)=1+7=8=2*k

Always (odd number)+(odd number)=even number. The point to note is that if a prime number has an even exponent, its sigma will be odd:

σ(72)=1+7+7=15=2*k-1

So, (p2*k-1)=2*k , σ(p2*k)=2*k-1 . Thus, Euler's concept came into being. On the right side of this

σ(p1a) * σ(p2b) * … * σ(pnz)=2*N formula, there is 2*N. This means that there should be only one factor of two on the left side. Because if there were two factors of two, it would become

σ(p1a) * σ(p2b) * …*σ(pnz)=4*k=2*N This shows that (4*k=2*N) N is even.1

From this, it follows that only one of the sigmas can be even:

N=(pj2*k-1) *(p1a * p2b * ... * pnz)

Then, Euler refined the formula and proved that it takes the form of : N=(p4*k+1)*(M2) However, he did not prove whether these numbers exist or not.

In 1644, Marn Mersenne examined numbers in the form of Euclid's formula, found the first 11 values of π, and claimed that these were the prime numbers.

2, 5, 7, 13, 17, 19, 31, 67

However, he acknowledged that he had not verified whether large numbers like

267-1=147573953589676412927 were prime or not.

Scientists continued their research. Most of them started with the list of prime numbers proposed by Mersenne. In their list, 67 was included. Edouard Lucas proved that 267-1 is not a perfect number.

In 1952, Rafael Robinson created a computer program to find Mersenne prime numbers. Within ten months, the next five Mersenne primes and their corresponding perfect numbers were found1.

T/r		Mersenne prime number	Perfect number
13		2^{521}-1	$(2^{521}-1)* 2^{521}$
14		2^{607}-1	$(2^{607}-1)* 2^{607}$
15		2^{1279}-1	$(2^{1279}-1)*2^{1279}$
16		2^{2203}-1	$(2^{2203}-1)* 2^{2203}$
17		2^{2281}-1	$(2^{2281}-1)* 2^{2281}$

Continuously, Mersenne prime numbers were found through the computer.

T/r	Mersenne prime numbers	Number of rooms
18	2^{3217}-1	969
19	2^{4253}-1	1281
20	2^{4423}-1	1332

21	2^{9283}-1	2917
22	2^{9941}-1	2993
23	2^{11213}-1	3376

In 2017, scientists discovered the 50th Mersenne prime number. It was the largest known Mersenne prime at that time. In 2018, the 51st Mersenne prime number, M82589933=282589933-1 was discovered. It is currently the largest known Mersenne prime.

Infinite perfect numbers:

If there are infinite prime numbers, then there are also infinite perfect numbers.

CONCLUSION

Perfect numbers are an important topic that reminds us of the contributions of Euclid and Euler to mathematics. Their work laid the foundation for many researches in mathematical analysis, algebra, and number theory. Even today, mathematicians are striving to uncover the mysteries of perfect numbers. Mersenne prime numbers are so large and rare that finding them requires a lot of time and computer resources. In 1991, scientists determined that if an odd perfect number exists, it must be greater than 10300. New achievements have since increased this number to 10220 . Given how large the numbers are, it is unlikely that a computer will find them anytime soon.

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