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Construction Of Three-Variable High-Order Defferential Equation Polynomial Solutions By Combinatorics Method

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ABSTRACT

In this paper, we study how basic systems of polynomial solutions of a differential equation of high order with mixed derivatives of a function of three variables are constructed using combinatorial methods

KEYWORDS

Polynomial Solutions , Three-Variable High-Order , matrix of coefficients

INTRODUCTION

In the article polynomial solutions of this equation the basis system is built in a combinatorial way

$$\left(\frac{\partial^2}{\partial x_1 \partial x_2}+\frac{\partial^2}{\partial x_2 \partial x_3}+\frac{\partial^2}{\partial x_1 \partial x_3}\right)^N U(x_1,x_2,x_3)=0 \qquad (1)$$

Polynomial solutions of this equation can be written in these two ways [1-2, 4-6]

$$H^{n,p}_{0,\alpha_2}(x_1,x_2,x_3)=\sum_{i=0}^{n-2p-\alpha_2}\sum_{k=0}^{i}(-1)^i\binom{i+p}{p}\cdot\binom{i}{k}\cdot x_1^{i+p-k,!}\cdot x_2^{p+k+\alpha_2,!}\cdot x_3^{n-2p-i-\alpha_2,!}, \quad (2)$$

$\alpha_2=0,1,\dots,n-2p.,$

$$H^{n,p}_{\alpha_1,0}(x_1,x_2,x_3)=\sum_{i=0}^{n-2p-\alpha_1}\sum_{k=0}^{i}(-1)^i\binom{i+p}{p}\cdot\binom{i}{k}\cdot x_1^{i+p-k+\alpha_1,!}\cdot x_2^{p+k,!}\cdot x_3^{n-2p-i-\alpha_1,!}, \quad (3)$$

$\alpha_1=0,1,\dots,n-2p.,$

In here $p=0,1,\dots,N-1;\ n\geq 2p$.

(2) and (3) in expressions if $\alpha_1=\alpha_2=0$ when polynomials fall from the top

$$H^{n,p}_{0,\alpha_2}(x_1,x_2,x_3)=H^{n,p}_{\alpha_1,0}(x_1,x_2,x_3)$$

For convenience $x_1^{a,!}\cdot x_2^{b,!}\cdot x_3^{c,!}$ we write factorial instead of a had.

We enter the concept of "indicator matrix" in this view

$$\begin{matrix}
 & & & & 0\,0\,n & & & & \\
 & & & 0\,1\,n-1 & & 1\,0\,n-1 & & & \\
 & & 0\,2\,n-2 & & 1\,1\,n-2 & & 2\,0\,n-2 & & \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
0\,n\,0 & & 1\,n-1\,0 & & \dots & & n-1\,1\,0 & & n\,0\,0
\end{matrix} \quad (4)$$

Similarly this

$$A^p_{i,k}=(-1)^i\binom{i+p}{p}\cdot\binom{i}{k} \quad (5)$$

We make a "matrix of coefficients" made of coefficients

$$A^p_{0,0}$$
$$A^p_{0,1} \qquad A^p_{1,0}$$
$$A^p_{0,2} \qquad A^p_{1,1} \qquad A^p_{2,0}$$
$$. \quad . \quad . \quad . \quad . \quad . \quad .$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$
$$A^p_{0,n-2p} \quad A^p_{1,n-2p-1} \quad . \quad . \quad . \quad A^p_{n-2p-1,1} \quad A^p_{n-2p,0}$$

The dimensions of these two matrices are the same.

These two matrices are superimposed, and we write the elements of the indicator matrix in degrees, and the elements of the coefficient Matrix as coefficients. The resulting polynomial (1) will be the solution of the equation [3].

We draw up all the combinations.

As an example $\alpha_1 = \alpha_2 = 0;\ n = 4;\ p = 0{,}1.$ let's look at the case.

“Indicator matrisa" will be in this view

$$0\,0\,4$$
$$0\,1\,3 \qquad 1\,0\,3$$
$$0\,2\,2 \qquad 1\,1\,2 \qquad 2\,0\,2$$
$$0\,3\,1 \qquad 1\,2\,1 \qquad 2\,1\,1 \qquad 3\,0\,1$$
$$0\,4\,0 \qquad 1\,3\,0 \qquad 2\,2\,0 \qquad 3\,1\,0 \qquad 4\,0\,0$$

“And quot; coefficients matrisa " takes this view

$$1$$
$$1 \quad 1$$
$$1 \quad 2 \quad 1$$
$$1 \quad 3 \quad 3 \quad 1$$
$$1 \quad 4 \quad 6 \quad 4 \quad 1$$

The first of these polynomials will be in this form

$$H^{4,0}_{0,0}(x_1, x_2, x_3) = x_3^{4,!} - x_2 x_3^{3,!} - x_1 x_3^{3,!} + x_2^{2,!} x_3^{2,!} + 2x_1 x_2 x_3^{2,!} + x_1^{2,!} x_3^{2,!} - x_2^{3,!} x_3 -$$
$$-3x_1 x_2^{2,!} x_3 - 3x_1^{2,!} x_2 x_3 - x_1^{3,!} x_3 + x_2^{4,!} + 4x_1 x_2^{3,!} + 6x_1^{2,!} x_2^{2,!} + 4x_1^{3,!} x_2 + x_1^{4,!}\ ;$$

The total number of these polynomials is 9 units, which (1) form the basis system of equation polynomial solutions.

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1) $H_{0,0}^{4,0}(x_1,x_2,x_3)=x_3^{4,!}-x_2x_3^{3,!}-x_1x_3^{3,!}+x_2^{2,!}x_3^{2,!}+2x_1x_2x_3^{2,!}+x_1^{2,!}x_3^{2,!}-x_2^{3,!}x_3-$

$-3x_1x_2^{2,!}x_3-3x_1^{2,!}x_2x_3-x_1^{3,!}x_3+x_2^{4,!}+4x_1x_2^{3,!}+6x_1^{2,!}x_2^{2,!}+4x_1^{3,!}x_2+x_1^{4,!}$;

2) $H_{0,1}^{4,0}(x_1,x_2,x_3)=x_2x_3^{3,!}-x_2^{2,!}x_3^{2,!}-x_1x_2x_3^{2,!}+x_2^{3,!}x_3+2x_1x_2^{2,!}x_3+x_1^{2,!}x_2x_3-x_2^{4,!}-3x_1x_2^{3,!}+3x_1^{2,!}x_2^{2,!}-x_1^{3,!}x_2$;

3) $H_{0,2}^{4,0}(x_1,x_2,x_3)=x_2^{2,!}x_3^{2,!}-x_2^{3,!}x_3-x_1x_2^{2,!}x_3+x_2^{4,!}+2x_1x_2^{3,!}+x_1^{2,!}x_2^{2,!}$;

4) $H_{0,3}^{4,0}(x_1,x_2,x_3)=x_2^{3,!}x_3-x_2^{4,!}-x_1x_2^{3,!}$;

5) $H_{0,4}^{4,0}(x_1,x_2,x_3)=x_2^{4,!}$;

6) $H_{1,0}^{4,0}(x_1,x_2,x_3)=x_1x_3^{3,!}-x_1x_2x_3^{2,!}-x_1^{2,!}x_3^{2,!}+x_1x_2^{2,!}x_3+2x_1^{2,!}x_2x_3+x_1^{3,!}x_3-x_1x_2^{3,!}-3x_1^{2,!}x_2^{2,!}-3x_1^{3,!}x_2-x_1^{4,!}$;

7) $H_{2,0}^{4,0}(x_1,x_2,x_3)=x_1^{2,!}x_3^{2,!}-x_1^{2,!}x_2x_3-x_1^{3,!}x_3+x_1^{2,!}x_2^{2,!}+2x_1^{3,!}x_2+x_1^{4,!}$;

8) $H_{3,0}^{4,0}(x_1,x_2,x_3)=x_1^{3,!}x_3-x_1^{3,!}x_2-x_1^{4,!}$;

9) $H_{4,0}^{4,0}(x_1,x_2,x_3)=x_1^{4,!}$.

These 9 polynomials are formed from placing the ends of the" indicator matrix "on the"matrix of coefficients".

Now, this means that some of the polynomial solutions to the equation (1,

$$\left(\frac{\partial^2}{\partial x_1\partial x_2}+\frac{\partial^2}{\partial x_2\partial x_3}+\frac{\partial^2}{\partial x_1\partial x_3}\right)U(x_1,x_2,x_3)=0 \qquad (2)$$

check when there are blind.

So, the first $H_{0,0}^{4,0}(x_1,x_2,x_3)=x_3^{4,!}-x_2x_3^{3,!}-x_1x_3^{3,!}+x_2^{2,!}x_3^{2,!}+2x_1x_2x_3^{2,!}+x_1^{2,!}x_3^{2,!}-$

$-x_2^{3,!}x_3--3x_1x_2^{2,!}x_3-3x_1^{2,!}x_2x_3-x_1^{3,!}x_3+x_2^{4,!}+4x_1x_2^{3,!}+6x_1^{2,!}x_2^{2,!}+4x_1^{3,!}x_2+x_1^{4,!}$ in the solution (2) $U(x_1,x_2,x_3)$ if we replace ni, then

$$\left(\frac{\partial^2}{\partial x_1\partial x_2}+\frac{\partial^2}{\partial x_2\partial x_3}+\frac{\partial^2}{\partial x_1\partial x_3}\right)H_{0,0}^{4,0}(x_1,x_2,x_3)=2x_3^{2,!}-3x_2x_3-3x_1x_3+4x_2^{2,!}+6x_1x_2+4x_1^{2,!}-x_3^{2,!}+$$

$$+x_2x_3+2x_1x_3-x_2^{2,!}-3x_1x_2-3x_1^{2,!}-x_3^{2,!}+2x_2x_3+x_1x_3-3x_2^{2,!}-3x_1x_2-x_1^{2,!}=0$$

we will have equality. It seems that the first solution (2) satisfied the equation.

If the second $H_{0,1}^{4,0}(x_1,x_2,x_3)=x_2x_3^{3,!}-x_2^{2,!}x_3^{2,!}-x_1x_2x_3^{2,!}+x_2^{3,!}x_3+2x_1x_2^{2,!}x_3+x_1^{2,!}x_2x_3-x_2^{4,!}-$

$-3x_1x_2^{3,!}+3x_1^{2,!}x_2^{2,!}-x_1^{3,!}x_2$ in the solution (2) $U(x_1,x_2,x_3)$ if we replace ni, this

$$\left(\frac{\partial^2}{\partial x_1\partial x_2}+\frac{\partial^2}{\partial x_2\partial x_3}+\frac{\partial^2}{\partial x_1\partial x_3}\right)H_{0,1}^{4,0}(x_1,x_2,x_3)=-x_3^{2,!}+2x_2x_3+x_1x_3-3x_2^{2,!}-3x_1x_2-x_1^{2,!}+x_3^{2,!}-x_2x_3-$$
$$-x_1x_3+x_2^{2,!}+2x_1x_2+x_1^{2,!}-x_2x_3+2x_2^{2,!}+x_1x_2=0$$

we will have equality. In this, too, (2) equality became zero. Or third

$H_{0,2}^{4,0}(x_1,x_2,x_3)=x_2^{2,!}x_3^{2,!}-x_2^{3,!}x_3-x_1x_2^{2,!}x_3+x_2^{4,!}+2x_1x_2^{3,!}+x_1^{2,!}x_2^{2,!}$ in the solution (2)

$U(x_1,x_2,x_3)$ if we replace ni, this

$$\left(\frac{\partial^2}{\partial x_1\partial x_2}+\frac{\partial^2}{\partial x_2\partial x_3}+\frac{\partial^2}{\partial x_1\partial x_3}\right)H_{0,1}^{4,0}(x_1,x_2,x_3)=-x_2x_3+2x_2^{2,!}+x_1x_2+x_2x_3-x_2^{2,!}-x_1x_2-x_2^{2,!}=0$$ we

will have equality. So this solution, like the above, (2) satisfied the equation.

We can continue this process in all solutions. Then we can see that all solutions (2) satisfy the equation.

In short, the construction of a three-dimensional mixed-product high-order differential equation polynomial solutions of the base system in a combinatorical way makes sense to find solutions to such seemingly differential equations.

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