The American Journal of Applied sciences
(ISSN – 2689-0992)
Published: May 31, 2021 | Pages: 160-171
Doi: https://doi.org/10.37547/tajas/Volume03Issue05-25

IMPACT FACTOR
2021: 5. 634
OCLC - 1121105553

Confidence Interval For An Unknown Mathematical Expectation Based On Weakly Related Observations

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ABSTRACT

In this article, we construct confidence intervals for estimating the unknown mathematical expectation of weakly related random variables. To do this, we first define a weak relationship.

KEYWORDS

Random Quantities , Algebra, Dimensional Function, Central Limit Theorem

INTRODUCTION

$\{X_n\}$ given a sequence of random quantities.

For this sequence, we define $\alpha -$ the coefficient of admixture as follows.

$$\alpha(k) = \sup_{n\geq 1} \sup_{A\in F_{-\infty}^{n}, B\in F_{n+k}^{\infty}} \left|P(AB) - P(A)P(B)\right| \to 0$$

Here, F_k^m , $X_k, \ldots, X_m$ $\sigma -$ that makes random quantities is algebra.

Definition: $\{X_n\}$ The sequence $\alpha -$ is said to satisfy the condition of mixture. If it is $\lim\limits_{k\to\infty} \alpha(k) = 0$

We consider that X_n s can be written in the following form. $X_i = \left(f(Y_{i+k})_{k\in Z} \right),\ i \geq 1$ Here

$f : R^z \to R -$ is a dimensional function $\{Y_n\}$, $\alpha -$

is a sequence that satisfies the condition of mixture.

According to the central limit theorem, the following relation is valid in terms of dependence for a stationary sequence.

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_i - \mu)\xrightarrow{D} N(0,\sigma^2)$$

Here

$$\sigma^2 = D(X_1) + 2\sum_{k=2}^{\infty} Cov(X_1, X_k)$$

$Cov(X_1, X_k) -$ covariance

With the aid of this theorem for μ reliability interval can be established. This requires a statistical evaluation of σ^2 .

Given a choice of $X_1,......X_n$ We devide this selection into blocks of equal length. We make the following definitions.

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_j, \quad \text{va} \quad \frac{1}{l}S_i(l) = \frac{1}{l}\sum_{j=(i-1)l+1}^{il} X_j,$$

According to the central limit theorem $\left(S_i(l) - l\mu\right)/\sqrt{l}$ normal $N(0,\sigma^2)$ approaches a random quantity sluggishly and $E\left|S_i(l) - l\mu\right| \to \sigma\sqrt{\frac{2}{\pi}}$. Instead of μ we put $\bar{X}_n$. We get the following mark as the mark of σ

$$\hat{B}_n = \frac{1}{\left[\frac{n}{l}\right]}\sqrt{\frac{\pi}{2}}\sum_{i=1}^{\left[\frac{n}{l}\right]}\frac{\left|S_i(l) - l\bar{X}_n\right|}{\sqrt{l}} \qquad (1)$$

Here $k_n = [n / l_n]$ is the number of blocks. (1)

Validity of the mark is proved.

For σ^2 another estimate is studied in [4] the validity of the estimate introduced in [4] is proved.

$$\hat{B}_n^{(2)} = \frac{1}{\left[\frac{n}{l}\right]} \sum_{i=1}^{\left[\frac{n}{l}\right]} \left(\frac{|S_i(l) - lX_n|}{\sqrt{l}} \right)^2 \qquad i = 1, \ldots\ldots, \left[\frac{n}{l}\right] \qquad k = \left[\frac{n}{l}\right] \qquad (2)$$

Our goal is to study these estimates n, l, γ and what is the reliability interval for μ at different values of parameters.

In this case, we take random quantities $\{\xi_i\}$ that are independent and and have a standard normal distribution and with the formulation of $X_n = \alpha X_{n-1} + \xi_i$ we find X_n. sequence $\alpha -$satisfies the condition of mixing.[5]. Through this sequence we learn (1) and (2) marks in different α, l and γ occasions . Thus, we find reasonable estimates of $X_n, S(l)$ and $\hat{B}_n$, $\hat{B}_n^{(2)}$ that is average value of the sample in order to construct the confidence intervals. Following this, we create confidence intervals for them. We use the following confidence intervals to create a confidence distance.

$$\left(\overline{x} - \frac{t_{\frac{1+\gamma}{2}}}{\sqrt{n}} \sigma; \overline{x} + \frac{t_{\frac{1+\gamma}{2}}}{\sqrt{n}} \sigma \right) \qquad (3)$$

Here σ is equal to the following.

$$\sigma = \hat{B}_n, \ \sigma = \sqrt{\hat{B}_n^{(2)}} \qquad (4)$$

The standard normal distribution quantities are shown in the table below.

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p	0.90	0.95	0.975	0.99	0.995	0.999	0.9995
U_p	1.282	1.645	1.960	2.326	2.576	3.090	3.291

$\xi_i \sim N(0,1)$ with a standard normal distribution $n=100$ selection is given in the following form.

0,926	-1,055	0,96	-0,918
-1,851	-0,162	-1,983	-0,725
-0,258	-0,345	-1,01	-0,199
0,161	0,181	-0,899	3,521
-1,501	0,856	-0,911	2,945
0,756	0,219	-0,472	-0,934
0,378	0,313	0,046	1,09
-1,229	1,393	0,595	0,748
-0,256	-0,963	0,971	1,372
0,415	1,394	0,303	1,678
1,096	-0,513	-0,457	1,356
1,375	0,007	-1,53	1,598
0,194	-0,136	-1,376	0,147
0,412	-0,511	0,598	-0,246
1,179	-0,736	0,012	0,571
-0,488	-0,491	1,231	1,974
-1,618	0,779	1,279	1,579
0,761	-0,005	0,321	-0,631
-0,486	-1,163	0,321	-0,69
-0,212	1,022	0,881	0,225
-0,169	-0,555	0,712	-0,057
0,181	-0,525	0,448	-0,561
0,785	0,769	-0,218	1,065
1,192	1,033	-0,482	-0,121
0,906	-2,051	-0,15	1,239

We find the confidence interval with the formula of (3) .With the formula (4) we calculate σ s.

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γ at the following results of reliability level $\gamma = 0,95$, $\gamma = 0.99$ we make reliability interval. In order to do this, we consider the following 3 cases.

$1-$**case** $n = 100$, $\alpha = 0.1$ and we make reliability intervals when the reliability levels are $\gamma = 0,95$, $\gamma = 0.99$ After some calculations, the selection of X_n is as follows.

X_n				X_100
0,926	-0,95168	0,765942	-0,93799	0,183292
-1,7584	-0,25717	-1,90641	-0,8188	
-0,43384	-0,37072	-1,20064	-0,28088	
0,117616	0,143928	-1,01906	3,492912	
-1,48924	0,870393	-1,01291	3,294291	
0,607076	0,306039	-0,57329	-0,60457	
0,438708	0,343604	-0,01133	1,029543	
-1,18513	1,42736	0,593867	0,850954	
-0,37451	-0,82026	1,030387	1,457095	
0,377549	1,311974	0,406039	1,82371	
1,133755	-0,3818	-0,4164	1,538371	
1,488375	-0,03118	-1,57164	1,751837	
0,342838	-0,13912	-1,53316	0,322184	
0,446284	-0,52491	0,444684	-0,21378	
1,223628	-0,78849	0,056468	0,549622	
-0,36564	-0,56985	1,236647	2,028962	
-1,65456	0,722015	1,402665	1,781896	
0,595544	0,067202	0,461266	-0,45281	
-0,42645	-1,15628	0,367127	-0,73528	
-0,25464	0,906372	0,917713	0,151472	
-0,19446	-0,46436	0,803771	-0,04185	
0,161554	-0,57144	0,528377	-0,56519	
0,801155	0,711856	-0,16516	1,008481	
1,272116	1,104186	-0,49852	-0,02015	
1,033212	-1,94058	-0,19985	1,236985	

When it is $l = 5$, we construct reliability intervals.

This process indicates that the reliability interval is shorter in $\hat{B}_n^{(2)}$ than at both levels of reliability level. Proof of this can be seen in the table below.

Sample size	n=100	
	l=5	
	k=20	
	0,1	
The average value of the sample.	0,183292	
B_100	1,378001	
B^2_100	0,884021	
reliability level1	0,95	
reliability interval1	T_1=(-0,0868;0,45338)	B_100
	T_2=(-0,06669;0,433275)	B^2_100
reliability level 2	0,99	
reliability interval 2	T_1=(-0,17168;0,538265)	B_100
	T_2=(-0,14526;0,511841)	B^2_100

The table indicates the values of $\hat{B}_n$ and $\hat{B}_n^{(2)}$ The T_1, T_2 here represent the confidence intervals of two values of confidence level $\hat{B}_n$ and $\hat{B}_n^{(2)}$ that illustrates reliability range of estimates. (In this and subsequent cases) .

When it is $l = 10$ the reliability interval is as follows.

Reliability level in both value of γ , the confidence interval of $\hat{B}_n$ is shorter.

Sample size	n=100	
	l=10	
	k=10	
	0,1	
The average value of the sample	0,183292	
B_100	1,613253	
B^2_100	1,817589	
Reliability level 1	0,95	
Reliability interval1	T_1=(-0,13291;0,49949)	B_100
	T_2=(-0,17296;0,53954)	B^2_100
Reliability level2	0,99	
Reliability interval 2	T_1=(-0,23228;0,598866)	B_100
	T_2=(-0,28492;0,651503)	B^2_100

When it is $l = 20$ the result shows that the reliability interval of $\hat{B}_n$ is shorter in both value of reliability level of γ

Sample size	n=100		
	l=20		
Block length	k=20		
	0,1		
The average value of the sample	0,183292		
B_100	1,361597		
B^2_100	1,84298		
Reliability level 1	1,96		
Reliability interval1	T_1=(-0,08358;0,450165)	B_100	
	T_2=(-0,177930,544518)	B^2_100	
Reliability level2	0,99		
Reliability interval2	T_1=(-0,16746;0,53404)	B_100	
	T_2=(-0,29146;0,658046)	B^2_100	

$2-$ **case** In $n=100$, $\alpha=0.2$ we find the sample of X_n and when the reliability levels are i $\gamma=0.95$ and $\gamma=0.99$ we find reliability intervals

X_n					X-100
0,926	-0,81962	0,596305	-0,96802		
-1,6658	-0,32592	-1,86374	-0,9186		
-0,59116	-0,41018	-1,38275	-0,38272		0,204618
0,042768	0,098963	-1,17555	3,444456		
-1,49245	0,875793	-1,14611	3,633891		
0,457511	0,394159	-0,70122	-0,20722		
0,469502	0,391832	-0,09424	1,048556		
-1,1351	1,471366	0,576151	0,957711		
-0,48302	-0,66873	1,08623	1,563542		
0,318396	1,260255	0,520246	1,990708		
1,159679	-0,26095	-0,35295	1,754142		
1,606936	-0,04519	-1,60059	1,948828		
0,515387	-0,14504	-1,69612	0,536766		
0,515077	-0,54001	0,258776	-0,13865		
1,282015	-0,844	0,063755	0,543271		
-0,2316	-0,6598	1,243751	2,082654		

-1,66432	0,64704	1,52775	1,995531		
0,428136	0,124408	0,62655	-0,23189		
-0,40037	-1,13812	0,44631	-0,73638		
-0,29207	0,794376	0,970262	0,077724		
-0,22741	-0,39612	0,906052	-0,04146		
0,135517	-0,60422	0,62921	-0,56929		
0,812103	0,648155	-0,09216	0,951142		
1,354421	1,162631	-0,50043	0,069228		
1,176884	-1,81847	-0,25009	1,252846		

When it is $l = 5$ the reliability interval illustrates that at the both value of reliability level the reliability interval of $\hat{B}_n^{(2)}$ is short.

Selection number	n=100	
	l=5	
	k=20	
	0,2	
The average value of the sample	0,204618	
B_100	1,506249111	
B^2_100	1,373014971	
Reliability level1	0,95	
Reliability interval1	T_1=(-0,09060694;0,499842712)	B_100
	T_2=(-0,064493049;0,4737882)	B^2_100
Reliability level2	0,99	
Reliability interval2	T_1=(-0,183391885;0,59267657)	B_100
	T_2=(-0,149070771;0,558306542)	B^2_100

When it is $l = 10$ at both value of reliability level of γ the confidence interval of $\hat{B}_n$ is shorter.

Sample size	n=100	
	l=10	
	k=10	
	0,2	
The average value of the sample	0,204618	
B_100	1,77287	
B^2_100	2,10797	
Reliability level 1	0,95	
Reliability interval1	T_1=(-0,142865;0,552101)	B_100
	T_2=(-0,20854;0,61778)	B^2_100

Reliability level 2	0,99	
Reliability interval2	T_1=(-0,252074;0,66131)	B_100
	T_2=(-0,33502;0,747631)	B^2_100

When it is $l = 20$ the reliability interval is as follows.

This means that at both values of reliability level of γ the reliability interval of $\hat{B}_n^{(2)}$ is shorter.

Sample size	n=100	
	l=20	
	k=5	
	0,2	
The average value of the sample	0,204618	
B_100	1,971387	
B^2_100	1,64043	
Reliability level 1	0,95	
Reliability interval1	T_1=(-0,18177;0,59101)	B_100
	T_2=(-0,11691;0,526142)	B^2_100
Reliability level 2	0,99	
Reliability interval 2	T_1=(-0,30321;0,712447)	B_100
	T_2=(-0,21796;0,627193)	B^2_100

$3-$**case when it is** $n = 100, \alpha = 0.3$, we find X_n selections, and when it is $\gamma = 0.99$ and $\gamma = 0.95$, we make confidence intervals.

X_100				X_100
0,926	-0,65391	0,453333	-1,00614	0,23193
-1,5732	-0,35817	-1,847	-1,02684	
-0,72996	-0,45245	-1,5641	-0,50705	
-0,05799	0,045264	-1,36823	3,368884	
-1,5184	0,869579	-1,32147	3,955665	
0,300481	0,479874	-0,86844	0,2527	
0,468144	0,456962	-0,21453	1,16581	
-1,08856	1,530089	0,53064	1,097743	
-0,58257	-0,50397	1,130192	1,701323	
0,24023	1,242808	0,642058	2,188397	
1,168069	-0,14016	-0,26438	2,012519	

1,725421	-0,03505	-1,60931	2,201756	
0,711626	-0,14651	-1,85879	0,807527	
0,625488	-0,55495	0,040362	-0,00374	
1,366646	-0,90249	0,024109	0,569877	
-0,07801	-0,76175	1,238233	2,144963	
-1,6414	0,550476	1,65047	2,222489	
0,268579	0,160143	0,816141	0,035747	
-0,40543	-1,11496	0,565842	-0,67928	
-0,33363	0,687513	1,050753	0,021217	
-0,26909	-0,34875	1,027226	-0,05063	
0,100273	-0,62962	0,756168	-0,57619	
0,815082	0,580113	0,00885	0,892143	
1,436525	1,207034	-0,47934	0,146643	
1,336957	-1,68889	-0,2938	1,282993	

Reliability interval when it is $l = 5$.

The result indicates that at both values of reliability level of γ the reliability interval of $\hat{B}_n^{(2)}$ is shorter.

Sample size	n=100	
	l=5	
	k=20	
	0,3	
The average value of the sample	0,23193	
B_100	1,689195	
B^2_100	1,490634	
Reliability level1	0,95	
Reliability interval 1	T_1=(-0,09915;0,563012)	B_100
	T_2=(-0,07229;0,536151)	B^2_100
Reliability level2	0,99	
Reliability interval 2	T_1=(-0,20321;0,667067)	B_100
	T_2=(-0,1679;0,631764)	B^2_100

We see the reliability interval when it is $l = 10$.

The reliability interval of $\hat{B}_n$ is shorter at both values of reliability level .

Sample size	n=100	
	l=10	
	k=10	
	0,3	
The average value of the sample	0,23193	
B_100	1,393124	
B^2_100	1,4964	
Reliability level1	0,95	
Reliability interval 1	T_1=(-0,04112;0,504982)	B_100
	T_2=(-0,06136;0,525224)	B^2_100
Reliability level2	0,99	
Reliability interval2	T_1=(-0,12694;0,590799)	B_100
	T_2=(-0,15354;0,617403)	B^2_100

We see the reliability interval when it is. $l = 20$.

When reliability level is $\gamma = 0.99$ the reliability interval of $\hat{B}_n^{(2)}$ is shorter, but when confidence interval is $\gamma = 0.95$ the reliability interval of $\hat{B}_n$ is also shorter.

Sample size	n=100	
	l=20	
	k=5	
	0,3	
The average value of the sample	0,23193	
B_100	1,280277	
B^2_100	1,29447	
Reliability level1	0,95	
Reliability interval1	T_1=(-0,019;0,482864)	B_100
	T_2=(-0,02179;0,485646)	B^2_100
Reliability level2	0,99	
Reliability interval 2	T_1=(-0,2179;0,485647)	B_100
	T_2=(-0,10153;0,565386)	B^2_100

The overall result shows that at the following values of parameters $\alpha = 0.1, l = 10, \ l = 20$, $\alpha = 0.2$, $l = 10$ and $\alpha = 0.3, l = 10$ and at the following results of the reliability level: $\gamma = 0.95$,

$\gamma = 0.99$ the reliability interval of $\hat{B}_n$ is shorter Hence, the use of $\hat{B}_n$ in the values of parameters that is given above, gives good results.

$\alpha = 0.1, l = 5$ $\alpha = 0.2, l = 5, l = 20, \alpha = 0.3, l = 5$ and at the following values of reliability levels $\gamma = 0.95,$ $\gamma = 0.99$ the confidence interval of $\hat{B}_n^{(2)}$ is shorter. So, when creating a confidence interval at a given value of α va l the use of $\hat{B}_n^{(2)}$ indicates good result.

The reliability interval varies depending on the value of reliability level, that is, when it is $\alpha = 0.3$, $l = 20$ at the $\gamma = 0.95,$ value of reliability level, the confidence interval of $\hat{B}_n$ is shorter. But, when it is $\gamma = 0.99$ the reliability interval of $\hat{B}_n^{(2)}$ gives shorter results.

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