



## Reduction Of A Term In A Scientific Theory

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### ABSTRACT

The subject of this research is the concept of reduction in the logic and methodology of science. On the one hand, reduction is understood as a relationship between a term and its defining expression within a scientific theory, on the other hand, as a relationship between two theories. Since the expansion of the theory occurs due to the introduction of new terms into its vocabulary with the help of nominal definitions, reduction is an operation opposite to the definition: due to reduction, terms are removed from the dictionary of the theory. Moreover, the theory itself is defined in accordance with the set-theoretic approach as a class of sentences that are closed with respect to derivability. The novelty of the research lies in the fact that it examines the semantic and epistemological aspects of the formal definition of reduction. In particular, the explication of the reduction relation between the two theories is based on the concept of functional equivalence of theories. It has been established that the list of basic terms of the theory can only be specified conventionally. All terms introduced with the help of nominal definitions turn out to be reducible. Consequently, a distinctive feature of a theoretical term is the possibility of its reduction.

### KEYWORDS

Reduction, definition, theory, descriptive term, paradox of analysis, functional equivalence of theories, semantics, deducibility, set theory, epistemology.

## INTRODUCTION

The term is understood as a descriptive (non-logical) expression of a natural or artificial language, the meaning of which is a set of objects from the objective universe of the theory. In this case, we are not interested in the nature of objects. From the point of view of the set-theoretic approach generally accepted in modern logic, it is impossible to strictly distinguish between descriptive and logical terms, since both are defined through sets and operations on sets. However, a clear difference between the terms arises in the situations of their use. If logical terms ("and", "not", "is", "everyone", etc.) are used to combine parts of a sentence or several sentences, that is, they play a syntactic role, then descriptive terms ("number", "function", "Planet", "speed", etc.) provide the connection of sentences with the subject area, that is, they are used for semantic purposes. Therefore, the study of the reduction of a term in scientific theory can be considered a semantic study.

3. The concept of logical deducibility (provability), which is fundamental for the definition of a theory, is borrowed from the formal axiomatic system of logic by D. Hilbert and V. Ackermann [1, p. 53-54]. However, natural and resolutive logical calculi show that for the implementation of the logical conclusion of some theorems from others, the presence of axioms in the theory is not necessary, the rules of inference are sufficient. A more general concept of formal inference, defined through the relation of direct logical consequence, can be found, for example, in S. Kleene [6, p. 78]. Thus, the remarks made in this article refer not only to a narrow class of axiomatized deductive theories of logic and mathematics, but also to meaningful theories

of natural and humanitarian sciences, since the relation of logical deduction is universal.

4. The article uses the following designations:  $\forall$  - universal quantifier;  $\exists$  - existential quantifier;  $\cup$  - operator of union of sets;  $\rightarrow$  - implication;  $\&$  - conjunction;  $\leftrightarrow$  - equivalence (as a propositional connective);  $|$  = - sign of direct logical consequence.

## MATERIAL AND METHODS

Intuitively, reduction can be understood as the inverse of the definition. If, during the definition, a new term is added to the dictionary of the theory, then during the reduction this term is removed from the dictionary, and in all expressions containing it, the eliminated term is replaced by some equivalent expression. The epistemological meaning of the reduction procedure is to find some irreducible (initial) terms. However, what properties should a term possess in order for its reduction to be possible? Let's try to answer this question using the criteria for the definition of a term in formal theory. At least four different criteria can be distinguished, but they are all logically related. 1. Explicit syntactic definability. The term P is explicitly syntactically defined in terms of the theory T if and only if there is a sentence S that does not contain the term P and includes k different variables  $x_1, x_2, \dots, x_k$ , such that from the set of sentences T the equivalence  $\forall x_1, x_2, \dots, x_k (P(x_1, x_2, \dots, x_k) \leftrightarrow S)$ . The criterion for explicit syntactic definability is not very useful, since its establishment requires finding a specific formula equivalent to the term P (for this reason, it is called explicit), and there is no effective general algorithm for such a search even for formal theories. 2. Implicit semantic

definability. The term  $P$  is implicitly semantically defined in terms of the theory  $T$  if and only for any semantic interpretations of  $I_1$  and  $I_2$  on some non-empty subject area  $U$  it is true that if they assign identical meanings to each term from the dictionary of the theory  $T$ , then they also assign identical meanings to the term  $P$ . Here, a semantic interpretation is understood as a certain function that assigns to each constant of the theory  $T$  a unique value from the subject area  $U$ . The criterion of implicit semantic definability was first proposed by the Italian logician A. Padoa [18], who showed that if the term of a theory is defined through its other terms, then it is not independent and, therefore, can be eliminated from its vocabulary. According to Padoa, the proof of the independence of a certain term in a theory from the rest of its original terms requires finding two such interpretations in which only the objects that are referents of this term are different. In other words, if two different semantic interpretations can be found for a term (functions that associate the same objects with all descriptive terms  $T$ , except  $P$ ), then it is an independent term. Padoa also proved that if a term  $P$  is explicitly syntactically definable in terms of a theory  $T$ , then  $P$  is implicitly semantically definable in terms of a theory  $T$ . This statement is known as Padoa's theorem and it is, in essence, analogous to the theorem on the semantic consistency of the predicate calculus [9, p. 51].

3. Implicit syntactic definability. Let  $P$  and  $Q$  be  $k$ -place terms belonging to the dictionary of the theory  $T$ . We denote by  $TQ$  the result of the widespread replacement of the term  $P$  by the term  $Q$  in  $T$ . We say that the term  $P$  is implicitly syntactically definable if and only if the formula  $\forall x_1, x_2, \dots, x_k (P(x_1, x_2, \dots, x_k) \leftrightarrow Q(x_1, x_2, \dots, x_k))$ .

4. Explicit semantic definability. The term  $P$  is explicitly semantically defined in terms of

the theory  $T$  if and only if there is a sentence  $S$  that does not contain the term  $P$  and includes  $k$  different variables  $x_1, x_2, \dots, x_k$ , such that the set of sentences  $T$  logically implies the equivalence  $\forall x_1, x_2, \dots, x_k (P(x_1, x_2, \dots, x_k) \leftrightarrow S)$ . Based on the above criteria, the Dutch logician E.V. Beth showed that if the term  $P$  is implicitly semantically definable in terms of the theory  $T$ , then it is implicitly definable (syntactically or semantically) in terms of the theory  $T$  [14]. This statement means that the implicit definability of a symbol in theory implies its dependence on other symbols. Initially, Beth's theorem was formulated only for predictor constants, but V.N. Karpovich proposed its generalized formulation for functional constants [3, p. 40-42]. Using the theorems of Padoa and Beth V.A. Smirnov demonstrated the equal volume of all four criteria for the definition of a term in the first-order predicate logic [9, p. 52-55]. Thus, the question of the existence of a definition of the term  $P$  in formal theory is reduced to the question of the existence of a derivation from the set of theorems of a given theory of some equivalence, in which  $P$  acts as a definition, and a sentence  $S$  that does not contain  $P$  as a definition. The main requirement for the definition operation is that the terms introduced with its help do not lead to the appearance of new unprovable sentences. Similarly, the main requirement for the reduction of a term should be that this operation does not remove previously proved theorems from the theory. Nevertheless, the introduction of a new term can be viewed as a transition from one theory to another, since the foundation of the theory - its vocabulary - is subject to change. From this point of view, the addition of any term to the theory  $T$  leads to the emergence of a new theory  $T'$ , any further change in the vocabulary  $T'$  leads to the

appearance of  $T$  ", etc. To avoid such unnecessary multiplication of theories, we will say that the theory  $T_1$  is an extension of the theory  $T_2$  if and only if any sentence  $S$  belonging to  $T_1$  is deducible from  $T_2$ . If the converse theorem is also true (any sentence  $S$  belonging to  $T_2$  is deducible from  $T_1$ ), then  $T_1$  is called a conservative extension of the theory  $T_2$ . V.A. Smirnov proved that a set of sentences plays the role of a definition of a term in formal theory if and only if the extension of the theory due to this set is a conservative extension [9, p. 58-60]. Now we can more strictly define the concept of term reduction in formal theory. A set of sentences  $R$  is called a reduction of a  $k$ -place term  $P$  in a theory  $T$  if and only if: the term  $P$  is implicitly semantically defined in terms of the theory  $T$ ; the set  $T$  is a conservative extension of the set  $T \cup R$ . Thus, from the formal point of view, the criteria for the definition of a term are also criteria for its reducibility. Indeed, if a term is undefined in theory, then this is the original term and, therefore, it cannot be reduced (replaced by an equivalent formula) without violating the conservative extension conditions. If a term satisfies the criterion of implicit semantic definability, then, according to Beta's theorem, it is also syntactically definable in this theory. The latter means the ability to deduce from the theory such an equivalence in which a given term is compared with a certain definition, that is, an expression that can replace this term in any contexts without any significant changes in the semantics of the theory.

## RESULTS AND DISCUSSION

In the previous section, we considered formal criteria for the definition of a term and formulated with their help a formal definition of reduction. However, the term "definition" in

the philosophy of science does not always mean a formal operation (as, in fact, the term "reduction"). For example, sentences that are correct from the point of view of the Russian language: "I determined that the mass of  $X$  is greater than the mass of  $Y$ " and "I determined that  $X$  is a rigid body" do not indicate the introduction of new definitions, but rather state certain facts. In this context, the words "I determined" can be replaced by the expressions "I learned" or "I was experimentally convinced". However, the sentence "I have defined mass as a scalar quantity expressing the gravitational and inertial properties of a body" contains a definition in the sense we need. Such definitions are called nominal. Nominal definitions (as opposed to "meaningful definitions") establish relationships between expressions of the language, not between objects of the domain. According to D.P. Gorsky: "A nominal definition is a definition by means of which: a) the meaning of a term already introduced into the language of science or into the natural language (including symbols in artificial languages of science) is formulated in an explicit form; b) the meaning of the newly introduced term in natural language or in the language of science is established ...; c) it is established that the terms defined ( $D_{fd}$ ) and defined ( $D_{fn}$ ) mean the same objects; d) a new term is introduced as a simple shorthand for another (usually more complex) expression. In this case, we temporarily abstract ourselves from the content of the terms  $D_{fd}$  and  $D_{fn}$  and consider only their sign forms "[2, p. 10-11]. Note that Gorsky uses the term "meaning" to describe properties (a), (b) and (c) of the nominal definition. It is the identity of meanings that is the main source of semantic problems of nominal definition and, consequently,

problems of reduction of terms. Indeed, the question is pertinent: why is the content of the concept of a nominal definition not limited to property (d)? Why do we want to understand the definition not just as the introduction of an abbreviation for a complex expression, but as a process of generating a new semantic entity? The result of numerous attempts to answer this complex philosophical question was the discussion around the so-called paradox of analysis, which unfolded in the second half of the 20th century. The paradox of analysis is formulated as follows: "If the verbal expression representing the analysandum has the same meaning as the verbal expression representing the analysans, then analysis simply establishes identity and is trivial; if these two verbal expressions do not have the same meaning, then the analysis turns out to be wrong" [4, p. 112]. For example, the statement "The concept 'brother' is identical with the concept 'male sibling'" has the same logical meaning as the statement "The concept 'brother' is identical with the concept 'brother'". However, the first statement is an informative definition of brother, while the second statement is nothing more than a trivial tautology. That is, from the point of view of classical extensional logic, these two statements are equivalent, but from the intensional point of view, they are not. There are various approaches to solving the paradox of analysis, but all of them, in our opinion, boil down to two main modes of reasoning: nominalist and realistic [12]. The nominalist approach fits well with formalism and logicism in metamathematics and with logical empiricism in philosophy. It is based on the conviction that deductive disciplines are artificial sign systems associated with their subject area only conventionally, and the theorems of logic and mathematics are purely

analytical constructions. The realistic way of reasoning proceeds from the objectivity of semantic information and considers language as a complex contextual field, not reducible to a set of formal rules. This direction includes intuitionism in metamathematics, the theory of deep and superficial information by J. Hintikka, neopragmatism and other philosophical trends that are critical in relation to logical empiricism. In both approaches, special attention is paid to information, since trivial identities ("a = a") differ from non-trivial definitions ("a = b") precisely in their information content. Since the 50s of the twentieth century, representatives of the nominalist approach are developing the theory of semantic information. For example, in the work of J. Bar-Hillel and R. Carnap "An Outline of a Theory of Semantic Information", the informativeness of a sentence is associated with the probability of its truth, while the subject area of this theory is limited to declarative language sentences (that is, the prescriptive level of the language is not considered) [fifteen]. The more likely a statement is, the less informative it is, and vice versa, the less likely a statement is, the more information it contains. In other words, the informative value of a judgment is inversely proportional to its probability. Thus, the most probable are analytical statements of the type "a = a" (that is, such sentences, the probability of truth of which is equal to one), but for the same reason they are the least informative for us. However, less probable synthetic statements like "a = b" (the probability of their truth only approaches one) have a certain degree of information content that can be expressed mathematically. In this context, such traditional philosophical concepts as "analytic" and "synthetic" acquire scientific significance. Another approach to the problem of information content, based on a realistic



way of reasoning, is proposed by J. Hintikka. He considers the well-known theorem of Boolean algebra, according to which any complex propositional function composed of atomic sentences can be associated with its disjunctive normal form (the construction of a disjunctive normal form is also possible for an arbitrary formula of predicate logic, but taking into account the quantifiers that can bind the variables in the formula) [17]. Hintikka notes that translating the language of formal theory into disjunctive normal forms gives us a tree-like structure, since each form can be represented as a graph, and the collection of graphs forms a tree of the language. The tree-like structure of the language allows you to characterize any sentence with the maximum number of "layers" of quantifiers contained in it. Hintikka calls the number of quantifiers in the scope of which the occurrence of a variable is the depth of the sentence. The depth of each propositional function is a computable characteristic, on the basis of which the concepts of deep and superficial information can be defined. When expanded into a tree structure, a synthetic sentence ( $a = b$ ) will receive a completely different depth than an analytic sentence ( $a = a$ ). Thus, deep information is a measure of the complexity of a sentence, or, in other words, a measure of the ability to expand a given sentence into a complex logical structure. The more complex the structure, the more informative the sentence, the more it contains semantic links with the subject area of the theory. Superficial information is defined as the probability of a propositional function without taking into account its internal structure. Hintikka explains the relationship between deep and superficial information as follows: "We can say that the deep information of a sentence is its surface information after we have processed this

sentence by all the means that logic puts at our disposal. Thus, surface information and deep information can be called, respectively, prelogical and postlogical information" [13, p. 167]. Both the nominalist and the realist approach solve the paradox of analysis, since they provide an opportunity to make an objective semantic assessment of a sentence and to distinguish trivial identities from nominal definitions. However, both approaches rely on the syntax and semantics of artificial languages and, therefore, are only partially applicable to theories stated in natural language. Semantic isolation, grammatical ambiguity and ambiguity of natural languages impede the effective solution of the analysis paradox and, therefore, make it impossible to clearly explicate the reduction. Here we are faced with a well-known dilemma: expressive possibilities versus the clarity and uniqueness of language. Nevertheless, for most metatheoretical studies, the proposed definition of reduction may be sufficient, since such studies concern mainly formal theories.

## CONCLUSION

Thus, the concept of reduction in logic and epistemology of science has two meanings: on the one hand, reduction can be understood as a relation between a term and its defining expression within a theory, on the other hand, as a relation between two theories. Following the set-theoretic approach, we understand theory as a class of sentences that are closed with respect to derivability. Since the expansion of the theory occurs due to the introduction of new terms into the dictionary with the help of nominal definitions, the reduction is an operation opposite to the definition: due to the reduction, terms are eliminated from the dictionary of the theory,

and the dictionary of the theory is narrowed. There are four main criteria for the definition of a term in scientific theory: implicit syntactic definability, explicit syntactic definability, implicit semantic definability, explicit semantic definability. If a term in a theory satisfies at least the criterion of implicit semantic definability, then this is a reducible term. Terms that are not definable in this theory are called irreducible (original) terms. Therefore, the reduction of a term can be defined as follows: it is a set of sentences  $R$  in a theory  $T$  such that (1) the term  $P$  is implicitly semantically defined in terms of the theory  $T$ ; (2) the set  $T$  is a conservative extension of the set  $T \cup R$ . However, the criteria for the definition of terms formulated for an artificially constructed language turn out to be ineffective in the transition to natural languages. Methods for resolving semantic paradoxes (for example, the theory of semantic information or the theory of deep and surface information) are to limit the language to declarative sentences. This is due to such properties of natural languages that impede the effective use of methods of logical formalization and analysis. The explication of the reduction relation between the two theories is based on the concept of functional equivalence of theories. Functionally equivalent theories are those theories that are identical in the function of deductive systematization of the chosen language. In symbolic logic, theorems are formulated and proved, asserting that for each deductively systematized theory containing auxiliary theoretical terms, it is possible to construct a theory that is functionally equivalent to it, containing only the original irreducible terms. Thus, we can say that two theories are related by a reduction relation if they are functionally equivalent and one of the theories contains only irreducible language

terms, and the second also contains auxiliary theoretical terms. In this case, a theory with theoretical terms can be called a reducible theory. The list of basic irreducible terms of the theory can only be specified conventionally. All terms introduced with the help of nominal definitions are called theoretical, or auxiliary, dependent, reducible. In this case, one does not have to talk about the empirical meaning of theoretical terms and assume their special ontological status. From this point of view, the only criterion that distinguishes an auxiliary term from a basic term is the possibility of its reduction from the dictionary of the theory.

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