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Some Estimates For The Carleman Function

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ABSTRACT

In this article we consider Carleman's functions, to find integral representation for the polygarmonious functions defined in unbounded domain of Euclidean space which Satisfies $2n \geq m$.

KEYWORDS

Soft drinks, antioxidant activity, herbal ingredients, green tea, local raw materials, multicomponent teas; coulometric analysis method; healthy food products.

INTRODUCTION

A task. Let be

$$u \in C^{2n}(D) \text{ and } \Delta^n u(y) = 0,\ y \in D \qquad (1)$$

$$u(y) = F_0(y),\ \Delta u(y) = F_1(y),\ \ldots,\quad \Delta^{n-1} u(y) = F_{n-1}(y),\ \ y \in S,$$

$$\frac{du(y)}{d\,\overline{n}} = G_0(y), \frac{d\Delta u(y)}{d\,\overline{n}} = G_1(y), \ldots, \frac{d\Delta^{n-1} u(y)}{d\,\overline{n}} = G_{n-1}(y), y \in S,$$

Where $F_i(y), G_i(y)$, $i = 0,1,\ldots, n-1$, given on S ($S \subset \partial D$) continuous functions, $\overline{n}$ - outer normal to ∂D needs to be restored $u(y)$ to D.

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We will assume that a solution to $u(y)$ problem (1)-(2) exists and $2n-1$ is continuously differentiable up to the endpoints of the boundary and satisfies a certain growth condition (correctness class), which ensures the uniqueness of the solution.

Functions $\Phi_\sigma(y,x)$ at $s>0, \sigma\geq 0, a\geq 0, 2n\geq m$ ($m-$ dimension space), we define:

$$\Phi_\sigma(y,x)=C_{n,m}\int_{\sqrt{s}}^{\infty}\mathrm{Im}\left[\frac{\exp\left(\sigma w+w^2\right)-achi\rho_1\left(w-\frac{h}{2}\right)}{\omega-x_1}\right]\left(u^2-s\right)^{n-k}du,\ \omega=iu+y_1 \quad (3)$$

$$C_{n,m}=(-1)^{\frac{m}{2}-1}\left(\Gamma(n-\frac{m}{2}+1)2^{2n-1}\pi^{\frac{m}{2}}\Gamma(n)\right)^{-1}$$

Theorem 1. The function $\Phi_\sigma(y,x)$ has

$$\Phi_\sigma(y,x)=C_{n,m}r^{n-k}\ln r+G_\sigma(y,x),$$

where it is $G_\sigma(y,x)$ regular in the variable y and continuously differentiable on $D\cup\partial D=\overline{D}$.

Evidence. In order , to get the statement of the theorem, consider the integrand

$$J_1=\mathrm{Im}\left[\frac{\exp\left(\sigma w-w^2\right)}{w-x_1}\right]$$

If we denote $y_1-x_1=\beta_1$ and bear in mind the properties of hyperbolic functions, we obtain

$$\exp(\sigma w+w^2)=\exp\left[\sigma(iu+\beta_1)+(iu+\beta_1)^2\right]=$$
$$=\exp\left[\sigma\beta_1+\beta_1^2-u^2\right]\exp i\left[\sigma u+2u_1^2\beta_1\right],$$

and then J_1 has the following form

$$J_1=\exp(\sigma\beta_1+\beta_1^2)\,\mathrm{Im}\frac{(\beta_1-iu)(\cos(\sigma u+2u\beta_1)+i\sin(\sigma u+2u\beta_1)}{(\beta_1-iu)(\beta_1+iu)\exp(u^2)}=$$
$$=\exp(\sigma\beta_1+\beta_1^2)\frac{\beta_1\sin(\sigma u+2u\beta_1)-u\cos(\sigma u+2u\beta_1)}{(\beta_1^2+u^2+s)\exp(u^2)}.$$

If a $J_1^1=\exp(\sigma\beta_1+\beta_1^2)\int_{\sqrt{s}}^{\infty}\frac{\beta_1\sin(\sigma u+2u\beta_1)}{(\beta_1^2+u^2+s)\exp(u^2)}\left(u^2-s\right)^{n-k}du$

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and $J_1^2 = \exp(\sigma\beta_1 + \beta_1^2)\int\limits_{\sqrt{s}}^{\infty} \frac{u\cos(\sigma u + 2u\beta_1)}{(\beta_1^2 + u^2 + s)\exp(u^2)}\left(u^2 - s\right)^{n-k} du,$

then

$$\Phi_\sigma(y,x) = C_{n,m}\left(J_1^1 - J_1^2\right) \quad C_{n,m} = (-1)^{\frac{m}{2}-1}\left(\Gamma(n - \frac{m}{2} + 1)2^{2n-1}\pi^{\frac{m}{2}}\Gamma(n)\right)^{-1}.$$

This is where our theorem follows.

Lemma 1. If a $\varphi_\sigma(y,x)$ harmonic function in R^m a variable y including a point x, then the equality

$$\Delta r^k \varphi_\sigma(y,x) = r^{k-2}\varphi_{\sigma,1}(y,x),$$

where the function $\varphi_{\sigma,1}(y,x) = (kn + k(k-2))\varphi_\sigma(y,x) + 2k\sum_{j=1}^{n}(y_j - x_j)\frac{\partial\varphi_\sigma(y,x)}{\partial y_j}$

is also a harmonic function in R^m the variable y including the point x.

Evidence. We have

$$\Delta r^k \varphi_\sigma(y,x) = knr^{k-2}\varphi_\sigma(y,x) + \sum_{j=1}^{m} k(y_j - x_j)\left(\frac{\partial r^{k-2}}{\partial y_j}\right)\varphi_\sigma(y,x)$$
$$+ 2k\sum_{j=1}^{m}\left(y_j - x_j\right)r^{k-2}\frac{\partial\varphi_\sigma(y,x)}{\partial y_j} + \sum_{j=1}^{m} r^k \frac{\partial^2\varphi_\sigma(y,x)}{\partial\ y_j^2}.$$

Since the $\varphi_\sigma(y,x)$ harmonic function in R^m the variable x, including the point y, then

$$k\sum_{j=1}^{m}(y_j - x_j)\frac{\partial r^{k-2}}{\partial y_j}\varphi_\sigma(y,x) = k\sum_{j=1}^{m}(k-2)(y_j - x_j)^2 r^{k-4}\varphi_\sigma(y,x =$$
$$= k(k-2)\sum_{j=1}^{m}(y_j - x_j)^2 r^{k-4}\varphi_\sigma(y,x) = k(k-2)r^{k-2}\varphi_\sigma(y,x)$$
$$\Delta r^k \varphi_\sigma(y,x) = (kn + k(k-2))r^{k-2}\varphi_\sigma(y,x) + 2kr^{k-2}\sum_{j=1}^{m}\left(y_j - x_j\right)\frac{\partial\varphi_\sigma(y,x)}{\partial y_j}$$

If a

$$\varphi_{\sigma,1}(y,x) = (kn + k(k-2))\varphi_\sigma(y,x) + 2k\sum_{j=1}^{m}(y_j - x_j)\frac{\partial\varphi_\sigma(y,x)}{\partial y_j}$$

Denoting, $c = (kn + k(k-2)) = k(n + k - 2)$, since is $\varphi_\sigma(y,x)$ harmonic with R^m respect to a variable y, including the point x, this implies the assertion of the lemma.

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Corollary 1. Under the conditions of Lemma 1, the equality holds $\Delta^k r^m \varphi_\sigma(y,x) = r^{m-2k}\varphi_{\sigma,k}(y,x)$, where the $\varphi_{\sigma,k}(y,x)$ harmonic function in R^m the variable y including the point $y = x$.

Corollary 2. The equalities are valid $\Delta r^k = k(k+m-2)r^{k-2}$,

$\Delta r^k \ln r = k(k+m-2)r^{k-2}\ln r + (2k+m-2)r^{k-2}$

Theorem 2. The function $\Phi_\sigma(y,x)$, defined using formula (3) is a polyharmonic function of order n with y respect to s> 0.

Theorem 3. For fixed $x \in D$ the function $\Phi_\sigma(y,x)$ satisfies

$$\sum_{k=0}^{n-1} \int_{\partial D \setminus S} \left[\left|\Delta^k \Phi_\sigma(y,x)\right| - \left| \frac{\partial \Delta^k \Phi_\sigma(y,x)}{\partial \bar{n}} \right| \right] ds_y \le C(x)\varepsilon(\sigma),$$

where the constant $C(x)$ is dependent on x and $\bar{n}$ - the outward normal to ∂D, $\varepsilon(\sigma) \to 0$ where $\sigma \to \infty$.

Corollary 3. The function $\Phi_\sigma(y,x)$, defined by formula (3) is the Carleman function for a point $x \in D$ and a part $\partial D \setminus S$.

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