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## Bisingular Integral In The Space Of Summable Functions

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### ABSTRACT

It is obtained a Zigmund type estimate for the bisingular integral in the space of Summation functions. It is constructed an invariant functional space based on the inequality. Using the method of successive approximations it is proven the solvability of the nonlinear bisingular integral equation in invariant space.

### KEYWORDS

Bisingular integral operator, Zigmund type estimate, invariant space, summable function.

### INTRODUCTION

The classical boundedness theorem of singular operator with the Hilbert kernel in space  $L_p$  ( $p > 1$ ), it was proved by N.H. Luzin in [6] and M.Riesz in [16] for the cases  $p=2$  and  $p > 1$ , respectively.

Subsequently, this result was carried over in a number of papers for fairly wide classes of Jordan rectifiable curves. A detailed prehistory

of this issue is available in the work [9], also in the works of A.P. Calderon [11], [12], and [13].

To study the special integral

$$\tilde{u}(x) = \int_a^b \frac{u(s)}{s-x} ds, \quad x \in (a, b)$$

$(-\infty < a < b < +\infty)$  with the summable density in the work [4], [10] for a function  $u \in L_p^{loc}(a, b)$ , where  $L_p^{loc}(a, b)$  - is the set of functions, summable with the

degree  $p$  in any compact segment of the interval  $(a, b)$ . The characteristics were introduced

$$\Omega_p(u, \xi, \eta) = \left( \int_{a+\xi}^{b-\eta} |u(x)|^p dx \right)^{\frac{1}{p}} \quad \xi, \eta > 0, \xi + \eta \leq b - a = l,$$

$$\omega_p(u, \delta, \xi, \eta) = \sup_{0 < h \leq \delta} \left( \int_{a+\xi}^{b-\eta-h} |u(x+h) - u(x)|^p dx \right)^{\frac{1}{p}}, \quad \xi + \eta + h \leq l, \delta > 0$$

and in the case  $1 < p < +\infty$  it is proved estimates,  $(\Omega_p(\tilde{u}), \omega_p(\tilde{u}))$ , by  $(\Omega_p(u), \omega_p(u))$ .

of an operator  $\tilde{u}$  in the space  $L_p(a, b)$  the results are obtained in [1], [2].

In the limiting case for  $p = \infty$  and  $u \in C[a, b]$  these results were obtained in [3], [7], it was shown that estimates [2] in a certain sense are unimprovable. In [5] using M. Riesz's theorem about the bounded action

One of the first papers, dedicated to the repeated special integral with the Hilbert kernel

$$(Bf)(x_1, x_2) = g(x_1, x_2) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x+t, y+\tau) \operatorname{ctg} \frac{t}{2} \operatorname{ctg} \frac{\tau}{2} dt d\tau$$

was a work of L. Cesari [14]. He proved that if  $f \in H_{(\delta_1^\alpha, \delta_2^\alpha)}^2$ , then

$$g \in H_{(\delta_1^\alpha |\ln \delta_1|, \delta_2^\alpha |\ln \delta_2|)}^2$$

Following L. Cesari, I.E. Zak [5] in his work also showed that the class of functions  $H_{((\delta_1^\alpha, \delta_2^\alpha))^2}$  is not invariant with

respect to the operator  $B$ . In the paper, it was proved that the classes of functions

$$H^{\alpha, \beta} = \left\{ f \in C_{[-\pi, \pi]^2} : \omega_f(\delta_1, \delta_2) = O(\delta_1^\alpha \delta_2^\beta), \omega_f^1(\delta_1) = O(\delta_1^\alpha), \omega_f^2(\delta_2) = O(\delta_2^\beta), 0 < \alpha, \beta < 1 \right\}.$$

are invariant with respect to the operator  
 B.

## RESULTS

Consider a bisingular integral of the form

$$\tilde{u}(x_1, x_2) = \int_{a_1}^{b_1} \int_{a_2}^{b_2} \frac{u(s_1, s_2)}{(s_1 - x_1)(s_2 - x_2)} ds_1 ds_2,$$

where the function  $u \in L_p^{loc}(\Delta)$ ,  $\Delta = (a_1, b_1) \times (a_2, b_2)$  and  $p > 1$ .

For the  $u \in L_p^{loc}(\Delta)$  we introduce the characteristic

$$\Omega_p(u, \xi_1, \eta_1, \xi_2, \eta_2) = \left( \int_{a_1 + \xi_1}^{b_1 - \eta_1} \int_{a_2 + \xi_2}^{b_2 - \eta_2} |u(x_1, x_2)|^p dx_1 dx_2 \right)^{\frac{1}{p}}.$$

$$\bar{\omega}_p(u, \delta_1, \xi_1, \eta_1, \xi_2, \eta_2)$$

$$= \sup_{h_1 \in E_1} \left( \int_{a_1 + \xi_1}^{b_1 - \eta_1 - h_1} \int_{a_2 + \xi_2}^{b_2 - \eta_2} |u(x_1 + h_1, x_2) - u(x_1, x_2)|^p dx_1 dx_2 \right)^{\frac{1}{p}}$$

$$\bar{\bar{\omega}}_p(u, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2)$$

$$= \sup_{h_2 \in E_2} \left( \int_{a_1 + \xi_1}^{b_1 - \eta_1} \int_{a_2 + \xi_2}^{b_2 - \eta_2 - h_2} |u(x_1, x_2 + h_2) - u(x_1, x_2)|^p dx_1 dx_2 \right)^{\frac{1}{p}}$$

$$\begin{aligned} &\omega_p(u, \delta_1, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2) \\ &= \sup_{h_1 \in E_1, h_2 \in E_2} \left( \int_{a_1 + \xi_1}^{b_1 - \eta_1 - h_1} \int_{a_2 + \xi_2}^{b_2 - \eta_2 - h_2} |\Delta u(x_1 + h_1, x_1, x_2 + h_2, x_2)|^p dx_1 dx_2 \right)^{\frac{1}{p}} \end{aligned}$$

where  $0 < \delta_i, \xi_i, \eta_i, \xi_i + \eta_i \leq l_i = b_i - a_i$ ,  $E_i = \{h_i : 0 < h_i \leq \min\{\delta_i, l_i - \xi_i - \eta_i\}, i=1,2$ .

$$\begin{aligned} \Delta u(x_1 + h_1, x_1, x_2 + h_2, x_2) &= u(x_1 + h_1, x_2 + h_2) - u(x_1 + h_1, x_2) - u(x_1, x_2 + h_2) \\ &+ u(x_1, x_2). \end{aligned}$$

Using [8], [15], it was proven

**Teopema 1.** Let  $1 < p < +\infty, \xi_i, \eta_i, \delta_i > 0, \xi_i + \eta_i \leq l_i, 0 < \delta_i \leq \min\{\xi_i, \eta_i\}$ . Then from convergence of correspondence integrals it holds the following inequality

$$\begin{aligned} \omega_p(\tilde{u}, \delta_1, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2) &\leq \\ &\leq C_p \left[ \frac{\delta_1 \delta_2}{(\delta_1 + \xi_1)(\delta_2 + \xi_2)} \frac{1}{(\xi_1 \xi_2)^{\frac{1}{q}}} \int_0^{\frac{\xi_1}{2}} \int_0^{\frac{\xi_2}{2}} \frac{\Omega_p\left(u, t_1, \frac{l_1}{2}, t_2, \frac{l_2}{2}\right)}{(t_1 t_2)^{\frac{1}{p}}} dt_1 dt_2 + \right. \\ &+ \left. \frac{\delta_1 \delta_2}{(\delta_1 + \xi_1)(\delta_2 + \eta_2)} \frac{1}{(\xi_1 \eta_2)^{\frac{1}{q}}} \int_0^{\frac{\xi_1}{2}} \int_0^{\frac{\eta_2}{2}} \frac{\Omega_p\left(u, t_1, \frac{l_1}{2}, \frac{l_2}{2}, t_2\right)}{(t_1 t_2)^{\frac{1}{p}}} dt_1 dt_2 + \right. \end{aligned}$$

$$\begin{aligned}
 & + \frac{\delta_1 \delta_2}{(\delta_1 + \eta_1)(\delta_2 + \xi_2)} \frac{1}{(\eta_1 \xi_2)^{\frac{1}{q}}} \int_0^{\frac{\eta_1}{2}} \int_0^{\frac{\xi_2}{2}} \frac{\Omega_p \left( u, \frac{l_1}{2}, t_1, t_2, \frac{l_2}{2} \right)}{(t_1 t_2)^{\frac{1}{p}}} dt_1 dt_2 + \\
 & + \frac{\delta_1 \delta_2}{(\delta_1 + \eta_1)(\delta_2 + \eta_2)} \frac{1}{(\eta_1 \eta_2)^{\frac{1}{q}}} \int_0^{\frac{\eta_1}{2}} \int_0^{\frac{\eta_2}{2}} \frac{\Omega_p \left( u, \frac{l_1}{2}, t_1, \frac{l_2}{2}, t_2 \right)}{(t_1 t_2)^{\frac{1}{p}}} dt_1 dt_2 + \\
 & + \frac{\delta_1}{(\delta_1 + \xi_1)} \frac{1}{\xi_1^{\frac{1}{q}}} \int_0^{\frac{\xi_1}{2}} \frac{\bar{\omega}_p \left( u, t_1, \frac{l_1}{2}, \delta_2, \frac{\xi_2}{2}, \frac{\eta_2}{2} \right)}{t_1^{\frac{1}{p}}} dt_1 \\
 & + \frac{\delta_2}{(\delta_2 + \xi_2)} \frac{1}{\xi_2^{\frac{1}{q}}} \int_0^{\frac{\xi_2}{2}} \frac{\bar{\omega}_p \left( u, \delta_1, \frac{\xi_1}{2}, \frac{\eta_1}{2}, t_2, \frac{l_2}{2} \right)}{t_2^{\frac{1}{p}}} dt_2 + \\
 & + \frac{\delta_1}{(\delta_1 + \xi_1)} \frac{1}{\eta_1^{\frac{1}{q}}} \int_0^{\frac{\eta_1}{2}} \frac{\bar{\omega}_p \left( u, \frac{l_1}{2}, t_1, \delta_2, \frac{\xi_2}{2}, \frac{\eta_2}{2} \right)}{t_1^{\frac{1}{p}}} dt_1 + \\
 & + \frac{\delta_2}{(\delta_2 + \xi_2)} \frac{1}{\eta_2^{\frac{1}{q}}} \int_0^{\frac{\eta_2}{2}} \frac{\bar{\omega}_p \left( u, \delta_1, \frac{\xi_1}{2}, \frac{\eta_1}{2}, \frac{l_2}{2}, t_2 \right)}{t_2^{\frac{1}{p}}} dt_2 + \\
 & + \omega_p \left( u, \delta_1, \frac{\xi_1}{2}, \frac{\eta_1}{2}, \delta_2, \frac{\xi_2}{2}, \frac{\eta_2}{2} \right)
 \end{aligned}$$

where the constant  $c_p$  is dependent on  $p$ .

Moreover, it is obtained estimates  $\Omega_p(\bar{u})$ ,  $\bar{\omega}_p(\bar{u})$ ,  $\bar{\omega}_p(\bar{u})$ . We denote by  $G$  the set of positive functions  $\varphi(\xi_1, \eta_1$

$[\xi_1, \xi_2], [\eta_1, \eta_2], (\psi) \bar{(\delta_1, \xi_1, \eta_1)}$   
 $[\xi_1, \xi_2], [\eta_1, \eta_2], \psi \bar{(\xi_1, \eta_1)}$   
 $[\delta_2, \xi_2], [\eta_2, \xi_2], \psi(\delta_1, \xi_1, \eta_1)$   
 $[\delta_2, \xi_2], [\eta_2, \xi_2]$   
 defined for  $\delta_i, \xi_i(i=1, 2)$   
 $\eta_i > 0, \xi_i(i=1, 2) + \eta_i = l_i = b_i - a_i$   $i=1, 2$ , and such

that the functions  $\varphi, \bar{\psi}, \bar{\bar{\psi}}$  almost decreasing in  $\xi_1, \xi_2$  (uniformly by other variables), the functions  $\psi, \bar{\psi}, \bar{\bar{\psi}}$  almost

increasing in  $\delta_1, \delta_2$  (uniformly by other variables), the functions

$$\frac{\bar{\psi}(\delta_1, \xi_1, \eta_1, \xi_2, \eta_2)}{\delta_1}, \frac{\bar{\bar{\psi}}(\xi_1, \eta_1, \delta_2, \xi_2, \eta_2)}{\delta_2}, \frac{\psi(\delta_1, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2)}{\delta_1}, \frac{\psi(\delta_1, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2)}{\delta_2},$$

almost decreasing in  $\delta_1, \delta_2$  (uniformly by other variables), last but not less

$$\bar{\psi}(\delta_1, \xi_1, \eta_1, \xi_2, \eta_2), \bar{\bar{\psi}}(\xi_1, \eta_1, \delta_2, \xi_2, \eta_2), \psi(\delta_1, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2) \rightarrow 0$$

for  $\delta_1, \delta_2 \rightarrow 0$ .

Let  $\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi \in G$ . Denote by  $H^P_{\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi}$  the set of functions from  $L^p_{loc}(\Delta)$  such that there exists constant  $c_i > 0$  ( $i = 1, 4$ ) and

$$\Omega_p(u, \xi_1, \eta_1, \xi_2, \eta_2) \leq c_1 \varphi(\xi_1, \eta_1, \xi_2, \eta_2),$$

$$\bar{\omega}_p(u, \delta_1, \xi_1, \eta_1, \xi_2, \eta_2) \leq c_2 \bar{\psi}(\delta_1, \xi_1, \eta_1, \xi_2, \eta_2),$$

$$\bar{\bar{\omega}}_p(u, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2) \leq c_3 \bar{\bar{\psi}}(\xi_1, \eta_1, \delta_2, \xi_2, \eta_2),$$

$$\omega_p(u, \delta_1, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2) \leq c_4 \psi(\delta_1, \xi_1, \eta_1, \delta_2, \xi_2, \eta_2).$$

The set  $H^P_{\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi}$  by norm  $\|u\|_{H^P_{\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi}} = \max\{c_1, c_2, c_3, c_4\}$  is a Banach space.

By  $G_0$  we denote the set of function  $\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi \in G$ , such that the following integrals are convergent

$$\int_0^{\frac{l_1}{2}} \int_0^{\frac{l_2}{2}} \frac{\varphi\left(t_1, \frac{l_1}{2}, t_2, \frac{l_2}{2}\right)}{(t_1 t_2)^{\frac{1}{p}}} dt_1 dt_2, \quad \int_0^{\frac{l_1}{2}} \int_0^{\frac{l_2}{2}} \frac{\varphi\left(t_1, \frac{l_1}{2}, \frac{l_2}{2}, t_2\right)}{(t_1 t_2)^{\frac{1}{p}}} dt_1 dt_2,$$

$$\int_0^{\frac{l_1}{2}} \int_0^{\frac{l_2}{2}} \frac{\varphi\left(\frac{l_1}{2}, t_1, t_2, \frac{l_2}{2}\right)}{(t_1 t_2)^{\frac{1}{p}}} dt_1 dt_2, \quad \int_0^{\frac{l_1}{2}} \int_0^{\frac{l_2}{2}} \frac{\varphi\left(\frac{l_1}{2}, t_1, \frac{l_2}{2}, t_2\right)}{(t_1 t_2)^{\frac{1}{p}}} dt_1 dt_2,$$

$$\int_0^{\frac{l_1}{2}} \frac{\bar{\psi}\left(t_1, \frac{l_1}{2}, \delta_2, \frac{\xi_2}{2}, \frac{\eta_2}{2}\right)}{t_1^{\frac{1}{p}}} dt_1, \quad \int_0^{\frac{l_2}{2}} \frac{\bar{\psi}\left(\delta_1, \frac{\xi_1}{2}, \frac{\eta_1}{2}, t_2, \frac{l_2}{2}\right)}{t_2^{\frac{1}{p}}} dt_2,$$

$$\int_0^{\frac{l_1}{2}} \frac{\bar{\psi}\left(\frac{l_1}{2}, t_1, \delta_2, \frac{\xi_2}{2}, \frac{\eta_2}{2}\right)}{t_1^{\frac{1}{p}}} dt_1, \quad \int_0^{\frac{l_2}{2}} \frac{\bar{\psi}\left(\delta_1, \frac{\xi_1}{2}, \frac{\eta_1}{2}, \frac{l_2}{2}, t_2\right)}{t_2^{\frac{1}{p}}} dt_2.$$

Now, we determine by  $H_p$  the set of positive functions

$\varphi(\xi_1, \xi_2), \bar{\psi}(\xi_1, \delta_2, \xi_2), \bar{\psi}(\delta_1, \xi_1, \xi_2), \psi(\delta_1, \xi_1, \delta_2, \xi_2,)$  satisfying the following conditions:

$$\int_0^{\frac{\xi_1}{2}} \int_0^{\frac{\xi_2}{2}} \frac{(t_1 t_2)^{\frac{1}{q}} \varphi(t_1, t_2)}{t_1 t_2} dt_1 dt_2 = 0 \left( (\xi_1 \xi_2)^{\frac{1}{q}} \varphi(\xi_1, \xi_2) \right),$$

$$\frac{\delta_1 \delta_2}{(\delta_1 + \xi_1)(\delta_2 + \xi_2)} \varphi(\xi_1, \xi_2) = 0 \left( \psi(\delta_1, \xi_1, \delta_2, \xi_2, ) \right),$$

$$\int_0^{\frac{\xi_1}{2}} \frac{t^{\frac{1}{q}} \bar{\psi}(t, \delta_2, \xi_2)}{t} dt = 0 \left( \xi_1^{\frac{1}{q}} \bar{\psi}(\xi_1, \delta_2, \xi_2) \right),$$

$$\frac{\delta_1}{\delta_1 + \xi_1} \bar{\psi}(\xi_1, \delta_2, \xi_2) = 0 \left( \psi(\delta_1, \xi_1, \delta_2, \xi_2, ) \right),$$

$$\int_0^{\frac{\xi_2}{2}} \frac{t^{\frac{1}{q}} \bar{\psi}(\delta_1, \xi_1, t)}{t} dt = 0(\xi_1^{\frac{1}{q}} \bar{\psi}(\delta_1, \xi_1, \xi_2)),$$

$$\frac{\delta_2}{\delta_2 + \xi_2} \bar{\psi}(\delta_1, \xi_1, \xi_2) = 0(\psi(\delta_1, \xi_1, \delta_2, \xi_2,)),$$

$$\psi\left(\delta_1, \frac{\xi_1}{2}, \delta_2, \frac{\xi_2}{2}\right) = 0(\psi(\delta_1, \xi_1, \delta_2, \xi_2,)),$$

$$\bar{\psi}\left(\delta_1, \frac{\xi_1}{2}, \xi_2\right) = 0(\bar{\psi}(\delta_1, \xi_1, \xi_2)),$$

$$\bar{\psi}\left(\xi_1, \delta_2, \frac{\xi_2}{2}\right) = 0(\bar{\psi}(\xi_1, \delta_2, \xi_2)),$$

where the constants in expression "0" do not depend on  $\delta_i, \xi_i (i = 1, 2)$ .

In a view of definition we say that  $(\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi) \in G_0 H_p$ , if  $(\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi) \in G_0$  and

$$\left(\varphi\left(\xi_1, \frac{l_1}{2}, \xi_2, \frac{l_2}{2}\right), \bar{\psi}\left(\delta_1, \xi_1, \frac{l_1}{2}, \xi_2, \frac{l_2}{2}\right), \bar{\bar{\psi}}\left(\xi_1, \frac{l_1}{2}, \delta_2, \xi_2, \frac{l_2}{2}\right), \psi\left(\delta_1, \xi_1, \frac{l_1}{2}, \delta_2, \xi_2, \frac{l_2}{2}\right)\right),$$

$$\left(\varphi\left(\frac{l_1}{2}, \eta_1, \xi_2, \frac{l_2}{2}\right), \bar{\psi}\left(\delta_1, \frac{l_1}{2}, \eta_1, \xi_2, \frac{l_2}{2}\right), \bar{\bar{\psi}}\left(\frac{l_1}{2}, \eta_1, \delta_2, \xi_2, \frac{l_2}{2}\right), \psi\left(\delta_1, \frac{l_1}{2}, \eta_1, \delta_2, \xi_2, \frac{l_2}{2}\right)\right),$$

$$\left(\varphi\left(\xi_1, \frac{l_1}{2}, \eta_2, \frac{l_2}{2}\right), \bar{\psi}\left(\delta_1, \xi_1, \frac{l_1}{2}, \eta_2, \frac{l_2}{2}\right), \bar{\bar{\psi}}\left(\xi_1, \frac{l_1}{2}, \delta_2, \eta_2, \frac{l_2}{2}\right), \psi\left(\delta_1, \xi_1, \frac{l_1}{2}, \delta_2, \eta_2, \frac{l_2}{2}\right)\right),$$

$$\left(\varphi\left(\frac{l_1}{2}, \eta_1, \frac{l_2}{2}, \eta_2\right), \bar{\psi}\left(\delta_1, \frac{l_1}{2}, \eta_1, \frac{l_2}{2}, \eta_2\right), \bar{\bar{\psi}}\left(\frac{l_2}{2}, \eta_1, \frac{l_2}{2}, \eta_2\right), \psi\left(\delta_1, \frac{l_2}{2}, \eta_1, \frac{l_2}{2}, \eta_2\right)\right) \in H_p$$

Теорема 2. If  $(\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi) \in G_0 H_P$ . Then operator  $\bar{u}$  maps the space  $H_0(\varphi, \bar{\psi}, \bar{\bar{\psi}}, \psi)$  into itself and is bounded.

We note that the proof of this last Theorem 2 comes from the proof of Theorem 1 and by definition of the sets of  $G_0 H_P$ .



## CONCLUSION

Using the method of successive approximations it is proven the solvability

of the nonlinear bisingular integral equation

$$u(x_1, x_2) = \lambda \int_0^{b_1} \int_0^{b_2} \frac{f(s_1, s_2, u(s_1, s_2))}{(s_1 - x_1)(s_2 - x_2)} ds_1 ds_2$$

in  $H_{\varphi, \bar{\varphi}, \bar{\psi}, \psi}^P$ , where the function  $f(s_1, s_2, u)$  is defined on  $(a_1, b_1) \times (a_2, b_2) \times (-\infty, +\infty)$  and  $\lambda$ - is a real parameter.

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